BIOMETRICS 45, 000–000
September 1989

A Stochastic Optimum Proportion Under a Shared System of Sole Crops

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SUMMARY

When responses on both components of a two-crop system are required, a shared system of sole crops is considered as an alternative to compare productivity of intercrops. The productivity of the shared system depends on the proportion of area alloted to the two sole crops. A method of choosing the shared proportion of the two sole crops in the same plot in a stochastic optimal way is presented in this paper. The probability of obtaining at least the specified yields on components and monetary returns have been compared for the shared system of sole crops and intercrops, using two practical examples.

1. Introduction

Intercropping is a system of growing two or more species (in separate rows) in conjunction on the same piece of land, whereas in mixed cropping seeds of species are mixed and sown in each row (Aiyer, 1949; Willey, 1979). Sole crops are often used to compare the productivity of intercrops or mixed crops. Sometimes only one type of crop species is compared with the intercrop. However, when both crops are required, a system called "shared system of sole crops" has been considered by Rao and Willey (1980). Under this system, a portion of the plot is sown under one crop, while the remaining part is allotted to the second crop. Thus, such a shared system consists of two sole systems arranged together in a plot. A shared sole-crop system differs from intercropping where the crop species are sown in alternate rows, or repeated in alternate sets of few rows. Intercropping is more difficult to cultivate compared with a shared system of sole crops. A diagram of the plot under three systems is given here.

Sorghum/Pigeonpea intercrop	Sole crops		Shared sole crops
	Sorghum	Pigeonpea	Sorghum Pigeonpea
SPSPSPSP	SSSSSSSS	PPPPPPPP	SSSPPPPP
SPSPSPSP	SSSSSSSS	PPPPPPPP	SSSPPPPP
SPSPSPSP	SSSSSSSS	PPPPPPPP	SSSPPPPP
SPSPSPSP	SSSSSSSS	PPPPPPPP	SSSPPPPP

In this example, S stands for sorghum and P for pigeonpea.

It may be noted that the proportion (θ) of the area for a crop, say S, may vary under the shared system, and hence a class of shared systems would be generated over the values of θ where $0 < \theta < 1$ ($\theta = 0$ or 1 results in a sole-crop system). For comparison with an intercrop system, an arbitrary choice [illegible] and the two crops are grown as sole crops in

Key words: Intercropping system; [illegible] probability integral; Shared system of sole crops.

f the plot. Depending on the requirement of the yield leve: one would vary the ... a desired crop. This raises the question as to the area to be allotted to a crop he shared system. Rao and Willey (1980) obtained the shared proportion θ such : shared system gives yields in proportion to that of the component yields of the p. But such a choice of θ depends on the competitive behaviour of the crop species in intercropping and might not be the most productive proportion if these crops are sown under the shared sole system, when the magnitude of competition arising between only the two rows, one row of each crop, might be negligible. Therefore, the need to obtain θ in an optimal sense described below arises and this paper attempts to develop a method to choose such an optimal θ.

Since intercropping and mixed cropping systems are practiced in environments where the weather is unpredictable or where it poses some risks for crops during crop growth. Rao and Willey (1980) compared various systems using probabilities of getting a specified net return. Such probabilities for getting specified yields on one component or both [considered by Pearce and Edmondson (1982)] avoid the effects of prices of crops. Mead et al. (1986) studied the joint distribution of net economic returns from sole crops and intercropping systems of sorghum, and evaluated their stability in terms of risk as probability of total return falling below the specified levels.

The choice of the shared proportion has been made employing the decision-making model based on the safety-first principle (Roumasset, Boussard, and Singh, 1979, p. 47, Chap. 3) and measures the risk in terms of the probability of the yields under a system (e.g., shared sole crops) falling below the specified values for any of the respective components. Here, an optimal θ is considered to be the one that minimizes the risk or, equivalently, maximizes the probability of obtaining at least the specified yields on the components. A comparison of an intercropping system with the optimal shared sole system using joint probabilities would be relevant. In the present paper, the shared system of sole crops has been assessed for riskiness, in terms of the actual productivity on the two crop components, instead of the total economic return given in Mead et al. (1986).

In Section 2, we present a method of determining the optimal θ and provide two illustrations on optimum values of θ. In Section 3, we present the joint probabilities from a shared system of the sole crops and intercropping systems.

2. Stochastic Optimum Shared Proportion

Under the sole system, it is reasonable to assume that the yields are normally distributed. Let the yield of crop S be normally distributed with mean μ_S and variance σ_S^2 while that of crop P has mean μ_P and variance σ_P^2. If a segment of an area θ of a unit area plot be allotted to sole crop S, then under the shared system, the yield on crop S, say Y_S, will follow $N(\theta\mu_S, \theta^2\sigma_S^2)$ and the yield on crop P, say Y_P, will be $N(\theta_1\mu_P, \theta_1^2\sigma_P^2)$, where $\theta_1 = 1 - \theta$. Further, let the minimum specified yields required under the shared system be $p\mu_S$ on crop S and $q\mu_P$ on crop P. The probability of obtaining at least these yields together is given by

$$Pr = \text{Prob}[Y_S \geq p\mu_S, Y_P \geq q\mu_P].$$

Furthermore, under the shared system, the yields of the two sole crops S and P are independent. Therefore, the above probability is simplified as below:

$$\begin{aligned} Pr &= \text{Prob}[Y_S \geq p\mu_S]\text{Prob}[Y_P \geq q\mu_P] \\ &= \text{Prob}[\theta(\mu_S + \sigma_S Z) \geq p\mu_S]\text{Prob}[\theta_1(\mu_P + \sigma_P Z) \geq q\mu_P] \\ &= \left\{1 - \Phi\left[\left(\frac{p}{\theta} - 1\right)C_S\right]\right\}\left\{1 - \Phi\left[\left(\frac{q}{\theta_1} - 1\right)C_P\right]\right\}. \end{aligned} \quad (2)$$

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The choice of the shared proportion has been made employing the decision-making model based on the safety-first principle (Roumasset, Boussard, and Singh, 1979, p. 47, Chap. 3) and measures the risk in terms of the probability of the yields under a system (e.g., shared sole crops) falling below the specified values for any of the respective components. Here, an optimal θ is considered to be the one that minimizes the risk or, equivalently, maximizes the probability of obtaining at least the specified yields on the components. A comparison of an intercropping system with the optimal shared sole system using joint probabilities would be relevant. In the present paper, the shared system of sole crops has been assessed for riskiness, in terms of the actual productivity on the two crop components, instead of the total economic return given in Mead et al. (1986).

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$$P_r = \text{Prob}[Y_S \geq p\mu_S, Y_P \geq q\mu_P).$$

Furthermore, under the shared system, the yields of the two sole crops S and P are independent. Therefore, the above probability is simplified as below:

$$\begin{aligned} P_r &= \text{Prob}[Y_S \geq p\mu_S]\text{Prob}[Y_P \geq q\mu_P] \\ &= \text{Prob}[\theta(\mu_S + \sigma_S Z) \geq p\mu_S]\text{Prob}[\theta_1(\mu_P + \sigma_P Z) \geq q\mu_P] \\ &= \left\{1 - \Phi\left[\left(\frac{p}{\theta} - 1\right)C_S\right]\right\}\left\{1 - \Phi\left[\left(\frac{q}{\theta_1} - 1\right)C_P\right]\right\}. \end{aligned} \qquad (2)$$

ystem was partitioned into variations due to fields and years within fields. The ntage variability accounted by fields ranged from 23% to 58% (58% for sole sorghum, or sorghum intercropped with pigeonpea, 26% for sole pigeonpea, 32% for pigeonpea intercropped with sorghum, 44% for pigeonpea intercropped with groundnut, 23% for sole dnut, and 49% for groundnut intercropped with pigeonpea). The variation between elds may be used for measuring the adaptability of the system, whereas the variability een years may reflect their instability.

r an evaluation of the riskiness of the systems, the part of the data involving only temporal effects would be needed. Thus, we attempted to control the effects of spatial variations by adjusting the yields for the field effects. The residuals so generated represent the temporal effects and were subsequently used to estimate the standard deviations (σ_1, σ_2) and correlation (ρ), presented in Table 1, required for computing the optimum shared proportion (θ).

The values of the optimum shared proportion (θ) for the two examples and for various choices of $p, q = .2, .4, .5, .6$, and $.8$ are presented in Table 2. The probabilities Prob[yield of first component $\geqslant p$ times its mean yield as sole crop, yield of second component $\geqslant q$ times its mean yield as sole crop] of obtaining specified minimum yields on respective components for the optimum shared system, and for intercrop systems, are also given in Table 2. The specified minimum yields are expressed in terms of means under respective sole crops for the sake of simplicity. The computation of the joint probability for intercrop was done using the expression for the bivariate normal probability integral given by Johnson

Table 2

Optimum shared proportion (θ), probability of obtaining more than specified component yields expressed in terms of p and q under shared system of two sole crops (PrSh) *and intercrops* (PrInt)

		Sorghum–Pigeonpea system			Pigeonpea–Groundnut system		
p	q	θ(sor)	PrSh	PrInt	θ(pp)	PrSh	PrInt
.2	.2	.44	.88	.92	.49	.76	.82
.2	.4	.36	.73	.78	.38	.61	.65
.2	.5	.33	.64	.68	.35	.53	.54
.2	.6	.31	.54	.56	.32	.45	.42
.2	.8	.27	.34	.31	.28	.30	.21
.4	.2	.61	.74	.88	.62	.61	[illegible]
.4	.4	.51	50	.75	.51	.42	.60
.4	.5	.48	.38	.65	.47	.34	.50
.4	.6	.45	.28	.54	.44	.26	.38
.4	.8	.41	.12	.30	.39	.14	.19
.5	.2	.67	.64	.84	.66	.53	.72
.5	.4	.57	.37	.71	.55	.33	.57
.5	.5	.53	.26	.62	.51	.25	.47
.5	.6	.51	.17	.51	.48	.18	.36
.5	.8	.46	.06	.29	.43	.06	[illegible]
.6	.2	.71	.52	.76	.69	.44	.67
.6	.4	.61	.25	.65	.58	.25	.52
.6	.5	.57	.16	.56	.55	.16	.42
.6	.6	.54	.09	.47	.52	.12	.33
.6	.8	.50	.03	.26	.47	.05	[illegible]
.8	.2	.76	.27	.50	.74	.28	[illegible]
.8	.4	.67	.08	.43	.64	.13	[illegible]
.8	.5	.63	.04	.38	.60	.08	.33
.8	.6	[illegible]	.02	.32	.57	[illegible]	[illegible]
.8	.8	.56	.00	.18	.52	.02	[illegible]

Note: sor = Sorghum, pp = Pigeonpea

and Kotz (1972, pp. 93–97). Table 2 exhibits that the probabilities under the intercrop system are higher than those under the optimum shared systems.

Although in this discussion the objective under a system has been specified in terms of yields, the optimum shared proportion can also be obtained for specified monetary return. Thus, for a minimum desired monetary return level *Mo*, the optimum shared proportion θ would be the one that maximizes the probability (of return exceeding *Mo* for the system)

$$\text{Prob}[\theta\lambda Y_S + (1 - \theta)Y_P \geq \alpha\mu_P] = 1 - \Phi(h),$$

where $h = [\alpha - (1 - \theta) - \lambda\theta r_1]C_P/[\theta^2\lambda^2 r_2 + (1 - \theta)^2]^{1/2}$, λ = ratio of the price of crop S to that of crop P, $Mo = \alpha\mu_P$ (specified return level as α times the return from the average crop P and expressed in terms of unit price of P), $r_1 = \mu_S/\mu_P$, and $r_2 = \sigma_S^2/\sigma_P^2$.

The computation of optimum θ was carried out for various meaningful combinations of price ratio (λ) and monetary levels (α) but most of these resulted in sole systems. A few cases are given in Table 3. The associated probabilities under intercrop were unity for these cases

Table 3

Optimum θ and probability of the minimum return under shared system (PrSh)

Case A Sorghum–Pigeonpea system				Case B Pigeonpea–Groundnut system			
α	λ	θ	PrSh	α	λ	θ	PrSh
.8	.3	.74	.79	.8	.5	.01*	.64
.8	.4	.81	.93	.8	.8	.24	.64
.8	.5	.81	.97	.8	1.2	.67	.79
1.0	.3	.99*	.51	.8	1.5	.86	.86
1.0	.5	.99*	.91				

* Resulting in sole crops.

A comparison of the shared system based on the optimum proportion with in[illegible] was made considering the minimum net return in the range of Rs 500 ha⁻¹ to Rs [illegible] ha⁻¹ for the crop combinations in Table 1. The prices Rs 2.40 kg⁻¹ for sorghum, Rs 6.40 kg⁻¹ for pigeonpea, and Rs 7.55 kg⁻¹ for groundnut used in the computation were for the week ending October 9, 1987, taken from the Weekly Wholesale Price Bulletin of Andhra Pradesh, Hyderabad (personal communication with Mr Parathasarathi Rao, RMP, ICRISAT). Based on sorghum–pigeonpea and pigeonpea–groundnut systems, we found that one would earn any minimum specified monetary return from intercropping systems [illegible] than from the shared system of sole crops based on an even optimum proportion [illegible] cases where minimum levels have been desired in terms of [illegible] sorghum [illegible] Table 2), the optimum shared systems of sole crops often resulted in sole systems when specified monetary returns exceeded approximately more than Rs [illegible] ha⁻¹.

ACKNOWLEDGEMENTS

The author thanks Dr M. R. Rao, sorghum agronomist, IITA, Nigeria, for helpful discussion [illegible] providing data for the illustration; Dr T. S. Walker, principal economist, ICRISAT [illegible] for constructive suggestions and Mr [illegible] P. Jaiswal and Mrs Leslie Pendleton Miller for secretarial assistance.

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Received August 1987; revised March and September 1988.