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A Stochastic Optimum Proportion Under a Shared System of Sole Crops

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SUMMARY

When responses on both components of a two-crop system are required, a shared system of sole crops is considered as an alternative to compare productivity of intercrops. The productivity of the shared system depends on the proportion of area allotted to the two sole crops. A method of choosing the shared proportion of the two sole crops in the same plot in a stochastic optimal way is presented in this paper. The probability of obtaining at least the specified yields on components and monetary returns have been compared for the shared system of sole crops and intercrops, using two practical examples.

1. Introduction

Intercropping is a system of growing two or more species (in separate rows) in conjunction on the same piece of land, whereas in mixed cropping seeds of species are mixed and sown in each row (Aiyer, 1949; Willey, 1979). Sole crops are often used to compare the productivity of intercrops or mixed crops. Sometimes only one type of crop species is compared with the intercrop. However, when both crops are required, a system called "shared system of sole crops" has been considered by Rao and Willey (1980). Under this system, a portion of the plot is sown under one crop, while the remaining part is allotted to the second crop. Thus, such a shared system consists of two sole systems arranged together in a plot. A shared sole-crop system differs from intercropping where the crop species are sown in alternate rows, or repeated in alternate sets of few rows. Intercropping is more difficult to cultivate compared with a shared system of sole crops. A diagram of the plot under three systems is given here.

Sorghum/Pigeonpea intercrop	Sole crops		Shared sole crops
	Sorghum	Pigeonpea	Sorghum Pigeonpea
SPSPSPSP	SSSSSSSS	PPPPPPPP	SSSPPPPP
SPSPSPSP	SSSSSSSS	PPPPPPPP	SSSPPPPP
SPSPSPSP	SSSSSSSS	PPPPPPPP	SSSPPPPP
SPSPSPSP	SSSSSSSS	PPPPPPPP	SSSPPPPP

In this example, S stands for sorghum and P for pigeonpea.

It may be noted that the proportion θ of the area for a crop, say S, may vary under the shared system, and hence a class of shared systems would be generated over the values of θ where $0 < \theta < 1$ ($\theta = 0$ or 1 results in a sole-crop system). For comparison with an intercrop system, an arbitrary choice of θ and $1 - \theta$ and the two crops are grown as sole crops in

Key words: Intercropping system, Stochastic probability integral, Shared system of sole crops

f the plot. Depending on the requirement of the yield level, one would vary the amount of a desired crop. This raises the question as to how much to be allotted to a crop in the shared system. Rao and Willey (1980) obtained the shared proportion θ such that the shared system gives yields in proportion to that of the component yields of the sole crops. But such a choice of θ depends on the competitive behaviour of the crop species in intercropping and might not be the most productive proportion if these crops are sown under the shared sole system, when the magnitude of competition arising between only the two rows, one row of each crop, might be negligible. Therefore, the need to obtain θ in an optimal sense described below arises and this paper attempts to develop a method to choose such an optimal θ .

Since intercropping and mixed cropping systems are practiced in environments where the weather is unpredictable or where it poses some risks for crops during crop growth. Rao and Willey (1980) compared various systems using probabilities of getting a specified net return. Such probabilities for getting specified yields on one component or both [considered by Pearce and Edmondson (1982)] avoid the effects of prices of crops. Mead et al. (1986) studied the joint distribution of net economic returns from sole crops and intercropping systems of sorghum, and evaluated their stability in terms of risk as probability of total return falling below the specified levels.

The choice of the shared proportion has been made employing the decision-making model based on the safety-first principle (Roumasset, Boussard, and Singh, 1979, p. 47, Chap. 3) and measures the risk in terms of the probability of the yields under a system (e.g., shared sole crops) falling below the specified values for any of the respective components. Here, an optimal θ is considered to be the one that minimizes the risk or, equivalently, maximizes the probability of obtaining at least the specified yields on the components. A comparison of an intercropping system with the optimal shared sole system using joint probabilities would be relevant. In the present paper, the shared system of sole crops has been assessed for riskiness, in terms of the actual productivity on the two crop components, instead of the total economic return given in Mead et al. (1986).

In Section 2, we present a method of determining the optimal θ and provide two illustrations on optimum values of θ . In Section 3, we present the joint probabilities from a shared system of the sole crops and intercropping systems.

2. Stochastic Optimum Shared Proportion

Under the sole system, it is reasonable to assume that the yields are normally distributed. Let the yield of crop S be normally distributed with mean μ_S and variance σ_S^2 while that of crop P has mean μ_P and variance σ_P^2 . If a segment of an area θ of a unit area plot be allotted to sole crop S, then under the shared system, the yield on crop S, say Y_S , will follow $N(\theta\mu_S, \theta^2\sigma_S^2)$ and the yield on crop P, say Y_P , will be $N(\theta_1\mu_P, \theta_1^2\sigma_P^2)$, where $\theta_1 = 1 - \theta$. Further, let the minimum specified yields required under the shared system be r_{μ_S} on crop S and q_{μ_P} on crop P. The probability of obtaining at least these yields together is given by

$$P = \text{Prob}\{Y_S \geq r_{\mu_S}, Y_P \geq q_{\mu_P}\}$$

Furthermore, under the shared system, the yields of the two sole crops S and P are independent. Therefore, the above probability is simplified as below

$$\begin{aligned} P &= \text{Prob}\{Y_S \geq r_{\mu_S}\} \text{Prob}\{Y_P \geq q_{\mu_P}\} \\ &= \text{Prob}\{\theta\mu_S + \sigma_S Z_1 \geq r_{\mu_S}\} \text{Prob}\{\theta_1\mu_P + \sigma_P Z_2 \geq q_{\mu_P}\} \\ &= \left\{ 1 - \Phi\left[\left(\frac{r_{\mu_S}}{\theta} - \mu_S\right)C_S\right] \right\} \left\{ 1 - \Phi\left[\left(\frac{q_{\mu_P}}{\theta_1} - \mu_P\right)C_P\right] \right\}, \end{aligned} \quad (2)$$

f the plot. Depending on the requirement of the yield level, one would vary the amount of a desired crop. This raises the question as to how much to be allotted to a crop in the shared system. Rao and Willey (1980) obtained the shared proportion θ such that the shared system gives yields in proportion to that of the component yields of the sole crops. But such a choice of θ depends on the competitive behaviour of the crop species in intercropping and might not be the most productive proportion if these crops are sown under the shared sole system, when the magnitude of competition arising between only the two rows, one row of each crop, might be negligible. Therefore, the need to obtain θ in an optimal sense described below arises and this paper attempts to develop a method to choose such an optimal θ .

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$$P_r = \text{Prob}\{Y_S \geq r_{\mu_S}, Y_P \geq q_{\mu_P}\}$$

Furthermore, under the shared system, the yields of the two sole crops S and P are independent. Therefore, the above probability is simplified as below

$$\begin{aligned} P_r &= \text{Prob}\{Y_S \geq r_{\mu_S}\} \text{Prob}\{Y_P \geq q_{\mu_P}\} \\ &= \text{Prob}\{(\mu_S - \sigma_S Z) \geq r_{\mu_S}\} \text{Prob}\{\theta_1(\mu_P + \sigma_P Z) \geq q_{\mu_P}\} \\ &= \left\{ 1 - \Phi\left[\left(\frac{r}{\theta} - 1\right)C_S\right] \right\} \left\{ 1 - \Phi\left[\left(\frac{q}{\theta_1} - 1\right)C_P\right] \right\}. \end{aligned} \quad (2)$$

system was partitioned into variations due to fields and years within fields. The nstage variability accounted by fields ranged from 23% to 58% (58% for sole sorghum, for sorghum intercropped with pigeonpea, 26% for sole pigeonpea, 32% for pigeonpea intercropped with sorghum, 44% for pigeonpea intercropped with groundnut, 23% for sole groundnut, and 49% for groundnut intercropped with pigeonpea). The variation between fields may be used for measuring the adaptability of the system, whereas the variability between years may reflect their instability.

In an evaluation of the riskiness of the systems, the part of the data involving only temporal effects would be needed. Thus, we attempted to control the effects of spatial variations by adjusting the yields for the field effects. The residuals so generated represent the temporal effects and were subsequently used to estimate the standard deviations (σ_1, σ_2) and correlation (ρ), presented in Table 1, required for computing the optimum shared proportion (θ).

The values of the optimum shared proportion (θ) for the two examples and for various choices of $p, q = .2, .4, .5, .6,$ and $.8$ are presented in Table 2. The probabilities Prob[yield of first component $\geq p$ times its mean yield as sole crop, yield of second component $\geq q$ times its mean yield as sole crop] of obtaining specified minimum yields on respective components for the optimum shared system, and for intercrop systems, are also given in Table 2. The specified minimum yields are expressed in terms of means under respective sole crops for the sake of simplicity. The computation of the joint probability for intercrop was done using the expression for the bivariate normal probability integral given by Johnson

Table 2
Optimum shared proportion (θ), probability of obtaining more than specified component yields expressed in terms of p and q under shared system of two sole crops (PrSh) and intercrops (PrInt)

p	q	Sorghum-Pigeonpea system			Pigeonpea-Groundnut system		
		θ (sor)	PrSh	PrInt	θ (pp)	PrSh	PrInt
.2	.2	.44	.88	.92	.49	.76	.82
.2	.4	.36	.73	.78	.38	.61	.65
.2	.5	.33	.64	.68	.35	.53	.54
.2	.6	.31	.54	.56	.32	.45	.42
.2	.8	.27	.34	.31	.28	.30	.21
.4	.2	.61	.74	.88	.62	.61	.77
.4	.4	.51	.50	.75	.51	.42	.60
.4	.5	.48	.38	.65	.47	.34	.50
.4	.6	.45	.28	.54	.44	.26	.38
.4	.8	.41	.12	.30	.39	.14	.19
.5	.2	.67	.64	.84	.66	.53	.72
.5	.4	.57	.37	.71	.55	.33	.57
.5	.5	.53	.26	.62	.51	.25	.47
.5	.6	.51	.17	.51	.48	.18	.34
.5	.8	.46	.06	.29	.43	.09	.18
.6	.2	.71	.52	.76	.68	.44	.67
.6	.4	.61	.25	.65	.58	.23	.52
.6	.5	.57	.16	.56	.55	.16	.43
.6	.6	.54	.09	.47	.52	.12	.33
.6	.8	.50	.03	.26	.47	.05	.17
.8	.2	.76	.27	.50	.74	.23	.43
.8	.4	.67	.08	.43	.64	.03	.41
.8	.5	.63	.04	.38	.60	.03	.33
.8	.6	.61	.02	.32	.57	.01	.28
.8	.8	.56	.00	.18	.52	.02	.17

Note: sor = Sorghum, pp = Pigeonpea

