

CP 306

Water Supply Optimization with Discrete Stochastic Linear Programming

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Abstract

A concept of improving irrigation management on Alfisols in hard-rock regions of the semi-arid tropics is presented. A discrete stochastic linear programming model was developed which permits the user to quantify the optimum water supply for composite watershed management for traditional agricultural production in south India. The model is flexible enough to be applicable to different site-specific conditions. Results show the beneficial impact of artificial groundwater recharging on agricultural production, employment and income. Open dug wells together with surface reservoirs for groundwater recharge form an economically sound system of water management for increasing productivity on a sustained basis.

Composite watershed management will substantially increase productivity of the watershed. It is likely to increase mainly paddy production but also oilseeds and wheat. It will also increase employment in rural areas. Further research, however, is needed to better understand the water flow from surface to aquifer and vice versa. Also improved systems of water management on farmers' fields are required especially for the irrigation of post-rainy-season crops.

An Alternative Concept of Water Management

Despite years of research at ICRISAT and other institutions, little progress has been made in improving traditional rainfed agriculture on Alfisols in the SAT. The water-retention capacity of Alfisols is too limited to permit effective management of monsoon rainfall within the soil profile. The need to find new ways of retaining rainfall, either on the surface or under the ground, for supplemental irrigation is now recognized. One traditional method of runoff management is to collect and store it in irrigation tanks. But a traditional tank (which occupies a large area with low efficiency) is difficult to maintain when population density increases and land becomes more valuable (von Oppen and Subba Rao 1980). Another method is well irrigation. But this type of irrigation often depletes groundwater. Alarming depletion of groundwater is reported from Tamil Nadu (Sivanappan and Aiyasamy 1978) and

other parts of the world. The negative effects of systems using groundwater on ecological balance, and the difficulties in managing surface-water irrigation projects (Bottrill 1981), call for exploring alternative approaches to irrigation management.

Hard-rock regions, especially granitic regions with single-layered aquifers in the SAT, seem suitable locations for trying out a different concept of irrigation management. This concept may be called composite watershed management (CWM). The concept is based on responsibilities shared by farmers and government: farmers manage their water sources efficiently and government authorities provide water sources and manage aquifers. Farmers do not generally even conceive of the problem of aquifer management (Stoner 1978). Geophysical conditions in these areas are such that lineaments (fractures and fissures) in the hard rock permit water distribution from the source to the fields. Advanced technologies make it possible to find these lineaments and put them to effective use.

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for locating wells and reservoirs (Todd 1980) Some of the villagers practices such as storing water in small reservoirs or canals, have benefited aquifer yields (Engelhardt 1983) Farmers in southern India close the sluice of tanks towards the end of the rainy season for sustained groundwater recharge. Some of these have been described by Baden-Powell (1892) for southern India, and by Dhavan (1981) for Sri Lanka. A policy for extending water-distribution systems over a large area to enhance artificial groundwater recharge has been suggested.

If focus of efforts to improve agriculture in the Alfisol regions of the SAT is shifted to composite watershed management two questions will arise

1. What is the impact of artificial recharging structures on water availability agricultural production employment and income?
2. What is the optimum size of reservoir for artificial recharging?

Both questions are related and call for simultaneous solution in order to arrive at an optimum investment strategy and optimum production organization. The most suitable tool for providing a solution to such a complex problem is a computerized mathematical model. Based on a field survey in a watershed near Hyderabad, a discrete stochastic linear programming model was designed. The results from different model runs are reported below. Only the agronomic and hydrological parameters obtained from field surveys and from literature were entered into the programming matrix. The infrastructural, economic, and natural conditions of a watershed, as well as the hydrologic properties of the aquifer, were not included as variables in the model. Because of site specificity there is no single answer to the problem of how to optimize water supply and allocation. Hence the solutions reported below relate to site-specific situations.

Model Description

Discrete stochastic linear programming model

Linear programming (LP) is a standard tool in economic planning. The disadvantage with ordinary LP models is that parameters and activities are assumed to be certain. Results of farm surveys, however, indicate the stochastic nature of agricultural production.

In the SAT the uncertainty of input and output levels is mainly due to unreliable rainfall. Consequently the model had to take into account the stochastic nature of rainfall and, hence, agricultural production. The discrete stochastic linear programming model developed for this research is shown diagrammatically in Figure 1.

The discrete stochastic linear programming (DSLPP) technique was first developed and applied by Rae (1971a and b) and used with simulation by Trebeck and Hardaker (1972). DSLPP differentiates between various states of nature. These states occur with probabilities. The probability of occurrence of one state of nature is based on past experience. All such activities as crop production, irrigation, marketing, etc., are formulated separately for every state. Parameters chosen are based on past experience.

In the study region rainfall in the rainy season is either high (>500 mm) or low (<400 mm). Since crop production takes place in two seasons (rainy season Jun-Nov and post-rainy season Dec-May) input-output relationships vary according to the amount and distribution of rainfall between the earlier (Jun-Sep) and the later part of the rainy season (Oct-Nov). (Note that the terminology for seasons used in this paper differs from more widely accepted definitions of rainy, post-rainy, and dry seasons.)

Following are the five different states for which input-output relationships had to be averaged.

For rainy-season cropping activities

- State 1 High rainfall years (probability of occurrence 0.5)
- State 2 Low rainfall years (probability of occurrence 0.5)

For post-rainy-season cropping activities

- State 3 High rainfall in early part and also high rainfall in later part ("wet after wet", probability of occurrence 0.375)
- State 4 High rainfall early and low rainfall late ("dry after wet", probability of occurrence 0.125)
- State 5 Low rainfall early and low rainfall late ("dry after dry", probability of occurrence 0.5)

(The state "wet after dry" does not occur.)

The different states are assumed not to compete for resources which are available in all states. But because rainfall determines the total water availability in a state, the activity levels in each state are

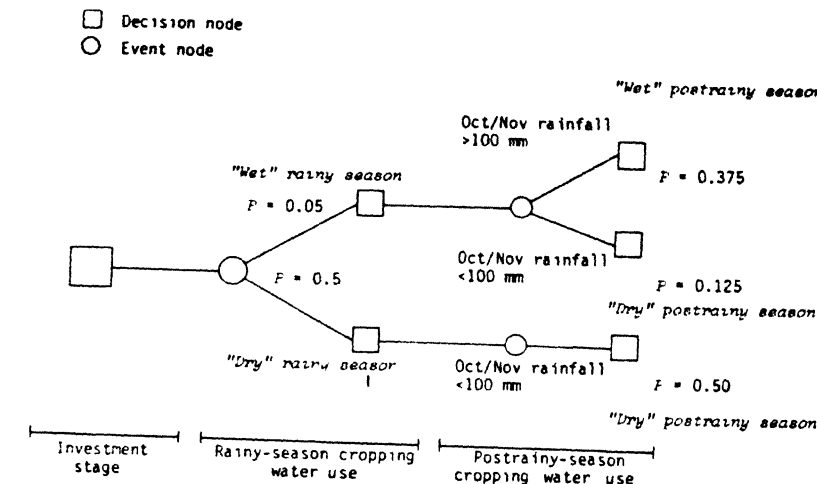


Figure 1 Diagrammatic representation of the discrete stochastic linear programming model

determined by water. The post-rainy and the rainy seasons, however, compete for water resources. Water can be used in the rainy season for irrigation or it can be transferred to the next season by storage. The objective function maximizes expected income from the different activities (i.e., income multiplied by probability of occurrence of a state). As a result, the optimum composition of activities related to the expected state will be obtained. The objective value is the weighted average of all activity levels that enter into the optimal solution.

There are several stages in the model. The first is the investment stage. During this stage, the decision whether to invest in reservoirs and wells is made based on expectations of occurrence of the various states. The extent of investment made in the first stage determines the resources available in the following stages and their respective states. Investment cost enters the objective function as annualized capital cost.

Water balance

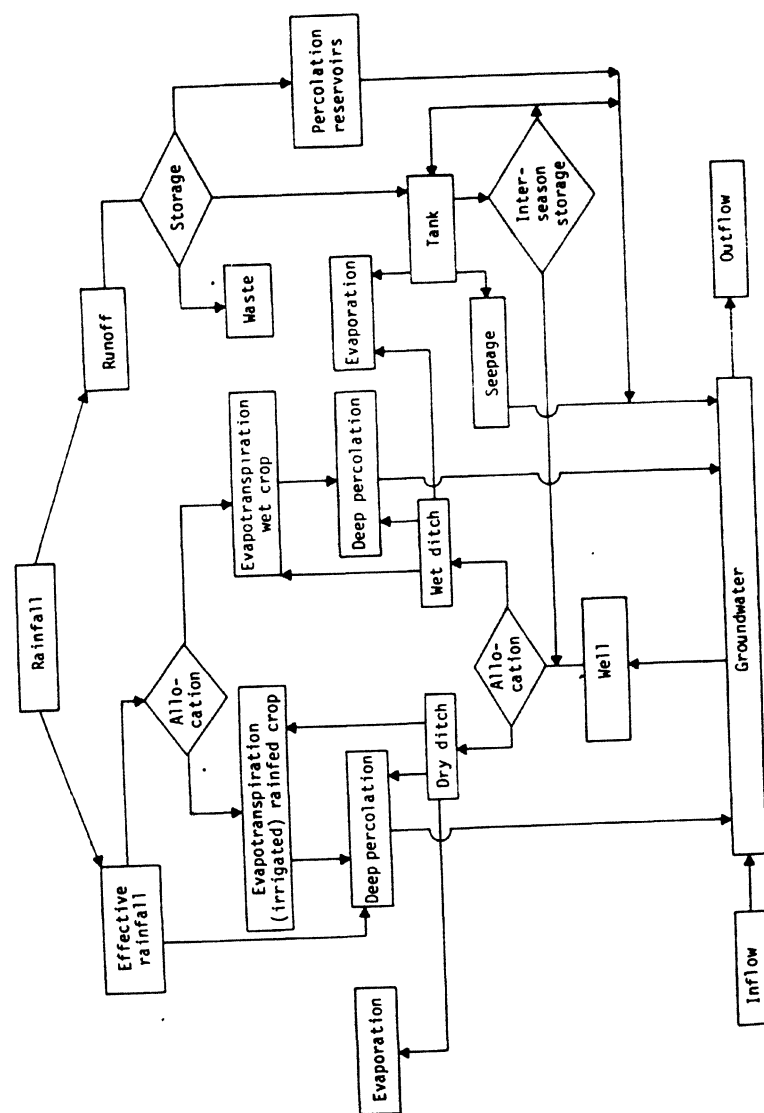
The model described above is combined with a water-balance model (Fig. 2). In this model the rain-

fall is divided into runoff and effective rainfall (the portion which infiltrates into the soil). Runoff can be stored in reservoirs that recharge groundwater. A certain percentage of effective rainfall recharges the aquifer directly by deep percolation. The remaining portion will be available to meet the evapotranspiration requirements of the crops. Because of the different agronomic characteristics, as far as water requirement is concerned, crops are differentiated into wet crops (e.g., paddy), and rainfed crops that can be provided with supplementary irrigation in times of drought stress.

Model runs and strategies

The model was used to optimize agricultural production and investment on percolation reservoirs for various economic settings. As shown in Figure 3, model runs were performed at two levels: (a) on an idealized 2000-ha region, and (b) on a 5-ha farm. The runs optimizing production in the two regions were divided into runoff less than 20% of rainfall, and runoff more than 20% of rainfall.

The latter represents a situation where only cropping and water-storage activities take place in the



Flow of the model on composite watershed development and management.

2000-ha watershed, but runoff is also brought in from outside in addition to runoff from within the region. Under this setting, land submerged by water competes with land for arable production, whereas total runoff is assumed to be unlimited.

Further model runs were made by varying the ratio of the submerged to the irrigated area. A possible strategy for implementing a percolation tank construction project could be to request every farmer who owns a well to sponsor one recharge reservoir of specific size. Runs in group 1 reflect such a strategy. Group 2 describes a situation where a maximum of 3.72% of the total watershed can be submerged.

In all groups of runs, two further sequences of runs were made, one describing a situation with old wells only (5, 10, and 15 wells per 100 ha) and in the other, investment was provided for new wells. The purpose of regional model runs was to define the optimum investment strategy for percolation reservoirs and wells. Depending on the cost of such reservoirs, and on the water demand in a region (i.e., where there are old wells, new ones can be constructed that have additional water requirements), various optimal compositions of hydrological infrastructures (wells, tanks) have been derived.

As the regional model is a "group-farm model", unlimited mobility of resources is assumed. This is not so in reality. Therefore results are not directly comparable with reality, which is more complex.

One run was made to assess a situation where a minimum paddy requirement had to be fulfilled. For the single farm, our interest concentrated on the impact of electricity cost on production and income, and on the profitability of well construction.

The Impact of Composite Watershed Management

Regional model

As the main concern is the impact of reservoir construction on production, labor, water use, and income, these parameters are reported for major runs (Table 1). The results are given as expected means, i.e., activity levels of the optimum solution are multiplied by the probabilities of occurrence of the states in which they are performed. It can be seen from Table 1 that pursuing a strategy of one reservoir per well will have little impact on production, employment, or water availability. If well density is

Model	a. Region	a1. Collected runoff < 20% of rainfall	0.1862 ha submerged/well	5 10 Wells/ 15 100 ha
			3.72% of total watershed submerged	5 10 Wells/ 15 100 ha
		a2. Collected runoff > 20% of rainfall	Optimum area submerged	5 10 Wells/ 15 100 ha
	b. Farm	Well capacity: 5 ha m ⁻³ a ⁻¹	Electricity price	0 : Rs kWh ⁻¹ 0.95
			Well investment cost	0 : Rs/well 50 000

Figure 3. Model runs.

Investment cost (Rs m ⁻²)	5 wells/100 ha (% submerged)	10 wells/100 ha (% submerged)	Net income (Rs ha ⁻¹)
0	13.7	28.7	111
0.28	11.9	25.0	117
0.87	10.5	25.0	120
0.96	8.5	18.1	121
1.01	5.2	8.2	122
1.18	1.5	5.6	123
4.13	1.0	4.0	124
5.51	0.0	0.0	125

low (5 wells/100 ha), the increase in net income per ha due to increase in storage volume is negligible. To submerge a larger area for the sole purpose of groundwater recharge is not profitable (at well density of 5 wells/100 ha and 13.7% arable area submerged), because groundwater availability is then no longer limiting production. However, as well density increases (e.g., 10 wells/100 ha) it becomes worthwhile to extend the area submerged.

Percolation reservoirs with wells could transform an unirrigated agricultural system into an irrigated system with a better submerged area to irrigated area ratio than that of traditional tank irrigation systems. It has been shown by von Oppen and Subba Rao (1980) that the ratio of command area to submerged area is 0.9 in traditional irrigation tank systems. In a model run of 5 wells/100 ha, this ratio is 1.8. For well density of 10 wells/100 ha the ratio would be 1.7.

In the case of 5 wells/100 ha, composite watershed management increases paddy production by 436%, supplementary irrigated oil seeds (e.g., groundnut) by 20%, and supplementary irrigated post-rainy-season crops (e.g., wheat) by 176%. Supply of sufficient water from percolation reservoirs would increase employment opportunities by 38%. The effect would be mainly on employment in the post-rainy season, which is traditionally a slack season. In this season, employment opportunities increase by 160%. But it is likely that construction of percolation reservoirs will lead to construction of more wells. Theoretically 33% of the arable land used should be submerged for maximum groundwater recharge. Wells may have to be licensed and discharge restricted by a quota system, or by imposing progressive water rates. It should also be clear that percolation reservoirs should be only part of a package of activities that include bunding, afforestation, and other practices.

The results reported above are based on runs with reservoir construction costs at zero. This was done assuming that costs and secondary benefits break even. The secondary benefits of percolation reservoirs on employment, siting of downstream reservoirs, water balance of the region, etc., are difficult to assess on purely economic grounds. Because benefits from one reservoir cannot be enjoyed exclusively by a single farmer, percolation tank construction can be pursued only by the community (e.g., government). Therefore construction costs are of secondary importance so long as benefits from the system outweigh the costs.

However, parametric changes in construction costs of percolation reservoirs can test the

of the optimum solution. Table 2 shows the optimum area submerged, depending on reservoir cost. This analysis shows that at zero cost of construction (to the farmer), a maximum area of 13.7% is brought under percolation tanks for 5 wells/100 ha, and 28.7% for 10 wells. As investment costs to the farmer increase, less and less area is submerged by percolation tanks. At present costs of about Rs 1 m⁻² only 5-8% of the area would be submerged.

The main result from this analysis is that percolation reservoirs are profitable if such benefits as employment generation, additional production and production stability are utilized in the analysis. It also shows that the cost of percolation tank construction cannot be borne by the farmers.

The model does not consider agronomic aspects of crop production. Therefore results should be carefully interpreted as far as the cropping pattern is concerned. For instance groundnut and wheat are two examples of irrigated dry crop production vs wet crops (e.g., paddy).

Interestingly, paddy production in the post-rainy season enters optimum solutions in almost every run. In addition, instead of groundnut, wheat is produced in the post-rainy season. Groundnut is mainly produced in the rainy season. Because agronomic constraints were not taken into account in the model, this result should not be overstressed.

Table 1 shows expected average cropping patterns, mean input quantities, and average production. These are based upon individual activities given by the model for every state. Because water constrains agricultural production in a dry post-rainy season following a dry rainy season, no production activities are performed. Instead, water is used in the rainy season. If water availability increases, production of hybrid paddy on a small area begins in the post-rainy season, and the limited amount of water is not spread over a larger area to irrigate dry crops.

Table 2. Optimum area submerged and cost of reservoir

Investment cost (Rs m ⁻²)	5 wells/100 ha (% submerged)	10 wells/100 ha (% submerged)
0	13.7	28.7
0.28	11.9	25.0
0.87	10.5	25.0
0.96	8.5	18.1
1.01	5.2	8.2
1.18	1.5	5.6
4.13	1.0	4.0
5.51	0.0	0.0

This is because paddy yields are high in a dry post-rainy season that follows a dry rainy season. Only further agronomic research can establish reasons for the differences in yield levels of irrigated crops during the post-rainy season. This example shows that input/output parameters of the same cropping activity vary according to different states of nature.

Tests were conducted with minimum paddy area per well. The minimum requirement was set at 1.4 ha for the rainy season and 0.9 ha for the post-rainy. These are average areas under paddy in the well command area. Results indicate that natural recharge from the catchment can sustain only 6 wells/100 ha under such requirements. With percolation reservoirs submerging 3.7% of the catchment, artificial and natural recharge sustains 24 wells/100 ha.

This result shows that, if a policy for increased paddy production in a small-holder irrigation system is to be pursued, percolation reservoirs are essential for safeguarding groundwater recharge. Natural recharge will normally be insufficient to recharge the aquifer if an increasing number of well owners irrigate paddy.

Farm model (5 ha)

The same model can be used at the regional or farm level. In the latter case provision has to be made for unlimited water supply because, for the individual farmer, the quantity of water that he can draw is limited only by the capacity of the well. The capacity of the well is defined by the properties of the geological formation of the strata into which it is sunk. Average well capacity in the region is about 30 000 m³ per season.

The model was used to optimize the cropping pattern for a model farm with or without a well, to determine the maximum cost of a well so that investing in it is profitable, and to compute the elasticities of water demand, depending on electricity rates. By comparing cropping results from a farm with well irrigation, and results from a farm without, the impact of irrigation on farm income, cropping pattern, employment, and factor-use has been assessed.

The expected net income (minus annual capital cost of the well) would be Rs 1117 ha⁻¹. The cropping pattern would provide employment for 1868 man-hours ha⁻¹ at a current wage rate of Rs 0.3 h⁻¹.

Without irrigation, all 5 ha of the farm could be planted with rainfed sorghum. This would provide 235 man-hours of employment per ha. The annual

expected net income from such dryland farming is Rs 192 ha⁻¹. A comparison of the two optimum solutions show that irrigation farming gives 5-6 times the expected net income per ha from pure rainfed farming.

The farm with irrigation facility would draw 45 000 m³ groundwater at an electricity tariff of Rs 0.16 kWh⁻¹. The cropping pattern is fairly stable regardless of electrical rate changes up to a certain level. It is when the electricity price exceeds Rs 0.6 kWh⁻¹ that changes in cropping activity may occur. This means that, for agricultural use, electrical rates could be increased to the rate charged for industry (Rs 0.6 kWh⁻¹) without affecting production. A price increase in electrical rates would, however, reduce income per ha. Table 3 shows the effect of electricity price (i.e., water price) on farm income and water consumption. It is likely that increased water rates will lead to more judicious water management, thereby counteracting the effect of reduced income.

It is possible to compute the average submerged area necessary to sustain the quantity of water consumed by the farm: 2.89 ha under a percolation reservoir is necessary to supply a 5-ha farm with sufficient water for optimal production. The ratio 1.73 of irrigated area to submerged area is much higher than that in traditional tank irrigation systems.

In the regional model, our interest focused on the impact of percolation reservoir costs on their feasibility. It was found that farmers may not voluntarily come forward to construct percolation reservoirs. Another constraining factor may be the cost of digging wells. Therefore, a run was done to compute the maximum amount a farmer will be able to invest in digging a well. As well costs increase solutions remain stable up to an investment of Rs 47 000 per well with characteristics as assumed in the model (capacity 30 000 m³ per season). This indicates that

farmers are likely to build their own wells because of the tremendous profitability of well irrigation as compared with unimproved rainfed farming.

Further Research Requirements

The results discussed above do not necessarily reflect reality because, besides water availability, other constraints such as labor capacity, crop rotations, and subsistence requirements were not considered. This exclusion was intentional, in order to ensure that water-resource constraints are not masked by other constraints.

There is need for further research to determine more precisely the dynamic nature of groundwater flow. Little progress has been made in research on this aspect in the past.

Research on agronomic aspects should cover water management at the farm level. More accurate measurements of water application at the field level should be made. This is mainly to understand better why farmers prefer to irrigate wet crops instead of providing supplemental irrigation for rainfed crops. From our point of view the tremendous impact on yield of small but strategic irrigation at critical growth stages should result in the wider adoption of supplemental irrigation. In field surveys farmers listed several factors (Engelhardt 1983), such as (a) excessive water, (b) subsistence requirements, (c) pest- and disease-control problems, and (d) marketing constraints. However, these factors do not appear to be plausible explanations considering the results from irrigation trials under research conditions.

As mentioned earlier, aquifer management is not new; it has been practiced for centuries and farmers take to it intuitively. Numerous aquifer irrigation projects have been implemented in India in the past, especially in the states of Andhra Pradesh, Tamil Nadu, and Maharashtra. There has, however, been no systematic monitoring of aquifer performance, farmers' response to this method of irrigation, and the impact on production. While there is a surprisingly large amount of knowledge on art groundwater recharge with various government departments, it is not readily available to those need it. ICRISAT could play an important part in collecting and disseminating this knowledge. Transfer of the concept of percolation tanks as wells to large areas in Africa with similar geological and climatic conditions seems feasible (Th 1982). More in-depth research, however,

on this aspect. Here again ICRISAT could play a key role in the transfer of a technology that not only could halt the dangerous lowering of the groundwater table, but holds out promises for improving agricultural production on SAT Alfisols.

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Table 3. Electricity rates and farm income of a model farm (5 ha, 1 well).

Rate Rs kWh ⁻¹	Annual net income (Rs ha ⁻¹)	Water pumped (m ³)
0.0	1442	64050
0.16	1117	45093
0.35	746	45093
0.50	450	45093
0.60	270	32816
0.95	192	0