On the role of information in decision making*

The case of sorghum yield in Burkina Faso

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Abstract: This paper investigates the role of temporal uncertainty and information issues in economic decisions. It shows that the nature of the economic environment (e.g. the production technology) can influence the valuation of information, which in turn affects the choice functions. This is illustrated by an empirical application to yield response analysis in Burkina Faso. The paper stresses the importance of technology and information valuation in risk behavior.

1. Introduction

The influence of risk on economic decisions has been the subject of much research. The most common approach has involved the modeling of 'timeless risk' in an expected utility framework. 'Timeless risk' is used here to refer to the case where either uncertainty is resolved immediately or all decisions are made at the same time. In this context, many attempts have been made to investigate and measure the timeless risk preferences of decision makers¹

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¹Timeless risk preferences are defined by the curvature of the von Neumann-Morgenstern utility function of a decision maker facing timeless risk [see Arrow (1965), Pratt (1964)].

[for examples in agriculture; see Anderson et al. (1977), Young (1979), Binswanger (1981)].

An alternative approach would be to consider that decisions are typically made sequentially in an uncertain environment. In this context, it is of interest to investigate the influence of temporal uncertainty on economic choices. Temporal uncertainty corresponds to the situation where decisions are made over time as new information becomes available resolving at least part of current uncertainty facing decision makers. This new information can influence economic decisions and the optimal allocation of resources. In particular, temporal risk generates 'induced risk preferences' over the temporal distribution of uncertainty [e.g. Spence and Seckhauser (1972), Drèze and Modigliani (1972), Kreps and Porteus (1979)]. Induced risk preferences depend on when uncertainty resolves. As argued by Drèze and Modigliani (1972), the risk premium for a delayed risk depends on two terms: (1) the value of information; and (2) the risk premium for the same timeless risk. But the value of information typically depends on the nature of the feasible set. This suggests that induced risk preferences can depend on the technology and the institutional environment of the decision maker. By implying that temporal risk behavior can be a by-product of the economic environment of the decision maker, it may help generate useful hypotheses about some of the factors influencing risk behavior. This could in turn help strengthen analysis of economic decisions under risk. For example, this could provide new insights on the role of uncertainty in agricultural development [e.g. Roumasset et al. (1978)]. Since temporal risk (as opposed to timeless risk) appears to be the rule rather than the exception, there is a need to refine our understanding of the role of temporal uncertainty in economic decision making.

The objective of this paper is to explore some of the implications of temporal uncertainty for risk behavior. We consider the case where current decisions are made under risk while other decisions are made in the future as uncertainty is being resolved. This allows an analysis of how technology and the temporal resolution of uncertainty can influence economic behavior under risk. It helps provide some insights on the impact of information and technology on economic decisions under uncertainty.

A general two-period model representing a decision-making process under temporal uncertainty is developed in section 2. It is argued that, in the absence of good a priori information about the nature of the objective function, the effect of timeless risk aversion on decisions may be difficult to identify under temporal risk. This is because the valuation of information and its impact on decisions may lead to behavior similar to the one obtained under timeless risk aversion. It suggests a need to better understand the role of information in economic analysis. A few examples are given in section 3 as illustrations of the usefulness of the approach. An application to yield response under temporal uncertainty is discussed in the context of an extended-quadratic production function (section 4). Some empirical evidence concerning the role of information valuation associated with rainfall uncertainty for sorghum production in Burkina Faso is presented in section 5.

2. A two-period model

Consider an economic agent facing a two-period planning horizon (t=1,2)and a preference function $U[w, x_1, x_2, e]$ where U is a twice continuously differentiable von Neumann-Morgenstern utility function satisfying $U_w = \frac{\partial U}{\partial w} > 0$, w is initial wealth, x_t is the vector of decision variables at time t, and e is a random vector representing temporal uncertainty about the state of the world. We assume that e is not known at time t=1 and has a given subjective probability distribution, but becomes observable by the agent costlessly before the second period decisions are made. In this context, if the agent maximizes expected utility, then economic decisions are made according to the following dynamic programming problem:

$$V(w) = \max_{x_1 \in X_1} E \max_{x_2 \in X_2} U[w, x_1, x_2, e],$$
(1)

where E is the expectation operator over the random variables e, X_1 is the feasible region for x_1 , $X_2(x_1)$ is the feasible region for x_2 , and V(w) denotes the indirect objective function of the agent. From backward induction, consider the second-period problem

$$\bar{U}(w, x_1, e) = \max_{x_2 \in X_2} U[w, x_1, x_2, e],$$

with $x_2^*(w, x_1, e)$ being the corresponding optimal choice function (conditional on x_1) Then, $\overline{U}(w, x_1, e)$ being the 'induced preference' function, the first-period problem takes the form

$$V(w) = \underset{x_1 \in X_1}{\operatorname{Max}} \operatorname{E} \overline{U}[w, x_1, e],$$

with $x_1^*(w)$ being the optimal first-period decisions. Note that $x_2^*(w, x_1, e)$ are expost decisions in the sense that they are made after e becomes observable, while $x_1^*(w)$ are ex ante decisions since they are made while e is still uncertain. If the decisions x_2 were made at time t = 1, they would correspond to the following problem:

$$\max_{x_2 \in X_2} EU[w, x_1, x_2, e],$$

which has for solution $\bar{x}_2(w, x_1)$ the ex ante choice functions for x_2 , which is conditional on x_1 .

From the literature on decision theory [e.g. LaValle (1978)], define the conditional value of information about e as the certain monetary value $D(x_1, w)$ which satisfies

$$E \operatorname{Max}_{x_2 \in X_2} U[w, x_1, x_2, e] = \operatorname{Max}_{x_2 \in X_2} E U[w + D(x_1, w), x_1, x_2, e],$$
(2)

or

$$EU[w, x_1, x_2^*(w, x_1, e), e] = EU[w + D(x_1, w), x_1, \bar{x}_2(w + D(x_1, w), x_1), e].$$

D is the amount of money that must be paid to the agent at time t=1 in order to make him indifferent between making the second-period decisions x_2 knowing e versus deciding about x_2 without learning about e. From (2), it follows that $D(x_1, w)$ is the conditional selling price of information about e: it is conditional on the first-period decisions x_1 ; and it is a selling price (or reservation price) in the sense that it measures the monetary value of doing away with the information, using the informed situation as the reference point. It is well known that the value of information is always non-negative,² i.e., that

$$D(x_1, w) \ge 0.$$

Indeed, obtaining costless information can never make a decision maker worse off. Furthermore, if the information is relevant to the decision-making process, obtaining it will usually make him better off. In this case, the agent would be made worse off by 'selling' relevant information and would need to be compensated by a lump-sum payment. This lump-sum payment $D(x_1, w)$ is the smallest amount of money which would make the agent willing to choose x_2 without learning about e. It represents the valuation of the ability to maintain flexible plans and revise decisions as new information becomes available.

The definition of the value of information in (2) is useful in the sense that it can be combined with (1) to reformulate the dynamic programming problem as the following timeless problem:

$$V(w) = \max_{\substack{x_1 \in X_1 \\ x_2 \in X_2}} EU[w + D(w, x_1), x_1, x_2, e].$$
(3)

²This follows from the assumption that $U_w > 0$ and the fact that $E \operatorname{Max}_{x_1} U(\cdot) \ge \operatorname{Max}_{x_2} EU(\cdot)$ [e.g. see LaValle (1978)]. Note that $D(x_1, w)$ is the gross value of information, which is always non-negative. However, when information is costly, then the *net* value of information $(D(\cdot) - C)$ can be positive, zero or negative depending on whether information cost C is less than, equal to, or greater than the gross value $D(\cdot)$.

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Denote by $\bar{x}_1(w)$ and $\bar{x}_2(w)$ the optimal solutions to (3). Clearly, $\bar{x}_1(w) = x_1^*(w)$ since (3) is simply a reformulation of (1): it corresponds to the ex ante decisions of the first period as before. However, $\bar{x}_2(w)$ is different from $x_2^*(w, x_1^*(w), e)$. First, it is now an ex ante decision since the decision $\bar{x}_2(w)$ is made based on the information available at time t=1 (i.e. before the agent learns about e). Second, $\bar{x}_2(w)$ is a compensated choice function since it can be influenced by the wealth compensation $D(\bar{x}_1, w)$. In other words, $\bar{x}_2(w)$ is the decision that would be made if the agent had to decide x_2 at time t=1 (i.e. before knowing e) while he is compensated for not being able to take advantage of the information that becomes available between t=1 and $t=2.^3$

Note that, with D set equal to zero, (3) would correspond to an open-loop solution, all decisions being made ex ante (before e is known). Given $\partial U/\partial w > 0$ and $D \ge 0$, this implies the well-known result that open-loop models are always inferior to closed-loop models [such as (1)] as they fail to capture the value of flexibility associated with the revision of plans [e.g. Bertsekas (1976, p. 204)]. This suggests that open-loop models of production are not appropriate tools of analysis whenever new information has a significant influence on economic decisions.

At this point, it will be useful to consider the special case where $U(w, x_1, x_2, e) = U(w + f(x_1, x_2, e))$, $f(\cdot)$ being the return function (e.g. discounted profit) and $[w + f(x_1, x_2, e)]$ representing the present value of terminal wealth. Then (3) takes the form

$$V(w) = \underset{\substack{x_1 \in X_1 \\ x_2 \in X_2}}{\text{Max EU}[w + D(w, x_1) + f(x_1, x_2, e)]}.$$
(4)

Using the Arrow-Pratt definition of the timeless risk premium,

$$R(w, x_1, x_2) = w + Ef(x_1, x_2, e) + D(w, x_1) - U^{-1}[EU(\cdot)],$$

expression (4) can be alternatively written in terms of its certainty equivalent as

$$\max_{\substack{x_1 \in X_1 \\ x_1 \in X_2 \\ x_1 \in X_2}} w + Ef(x_1, x_2, e) + D(w, x_1) - R(w, x_1, x_2).$$
(5)

³A special case of interest may be associated with the absence of a wealth effect in the decisions \bar{x}_2 , i.e. $\partial \bar{x}_2/\partial w = 0$ [e.g. the case of constant absolute risk aversion for a firm maximizing the expected utility of terminal wealth; see Chavas (1985)]. In this case, the compensation $D(x_1, w)$ will have no influence on \bar{x}_2 , and \bar{x}_2 would become the decision made if the agent had to decide x_2 at time t=1, i.e. \bar{x}_2 could be interpreted as the ex ante plans of the agent for x_2 .

As argued by Drèze and Modigliani, eq. (5) shows that the risk premium for temporal risk is $\overline{R} = D(w, x_1) - R(w, x_1, x_2)$, i.e., the difference between the conditional value of information $D(w, x_1)$ and the timeless risk premium $R(w, x_1, x_2)$. Note that, in the absence of risk D = R = 0, implying (as expected) a zero temporal risk premium.

Also, from eq. (5), it follows that the timeless risk premium R is positive under timeless risk aversion [where U(w) is a concave function], while R is zero under timeless risk neutrality [where U(w) is a linear function] [see Arrow (1965), Pratt (1964)].

Expression (5) indicates that current decisions are made by maximizing the sum of the three terms in (5). It suggests that it is desirable to have (a) a high expected return $Ef(\cdot)$, (b) a low risk premium $R(\cdot)$ if the agent is averse to timeless risk (R > 0), and (c) a high value of $D(\cdot)$, i.e. a good ability to adapt to temporal uncertainty. In the riskless case, R = D = 0, and (a) yields the familiar case of profit maximization. Characteristic (b) corresponds to the introduction of a non-linear utility function U(w) under risk in static models [e.g. Arrow (1965), Pratt (1964)]. Mostly in the context of timeless risk, an extensive amount of rersearch has focused on the influence of behavior on farmer's decisions [e.g. Anderson et al. (1977), Newbery and Stiglitz (1981), Binswanger (1981), Roumasset et al. (1978)]. Finally, characteristic (c) reflects the ability of the agent to modify production plans as new information becomes available. The issue of the valuation of information has received much attention in the literature [for agricultural examples, see Baquet et al. (1976), Byerlee and Anderson (1982), Bosch and Eidman (1987)]. However, little work has been done on the empirical implications of the conditional value of information $D(w, x_1)$ for technological choice and farm production decisions (as represented by the vector x_1). If flexibility of production and marketing plans is an important way for decision makers to deal with temporal uncertainty, then additional research on this topic may have high potential payoffs.

Note that D and R are clearly different since D tends to increase the value of the objective function while R would decrease it under timeless risk aversion. Also, under a linear utility function U(w), R=0 but, in general, $D \ge 0$. Thus, under temporal uncertainty $D \ge 0$ is not due to timeless risk preferences since the flexibility to respond to new information remains important even under a linear utility function.

To further illustrate this, consider the following *local* measure of the risk premium in (5) [Pratt (1964)]:

$$R = \frac{1}{2}\alpha \operatorname{Var}[f(\cdot)], \tag{6}$$

where $\alpha = -U_{ww}/U_w$ is the Arrow-Pratt absolute risk-aversion coefficient, Var(·) denotes the variance and subscript letters denote derivatives (e.g.

 $U_w = \partial U/\partial w$, $U_{ww} = \partial^2 U/\partial w^2$). Similarly, using the definition of the value of information, then a *local* measure of D for unconstrained problem (2) can be shown to be⁴

$$D = \frac{\text{Cov}\left\{U_{x_2}(\bar{x}_2), x_2^*\right\} + \frac{1}{2}\text{E}\left\{\left[x_2^* - \bar{x}_2\right]' \cdot U_{x_2x_2}(\bar{x}_2) \cdot \left[x_2^* - \bar{x}_2\right]\right\}}{\text{E}U_w(\bar{x}_2)} \ge 0.$$
(7)

A comparison of (6) and (7) indicates that, although R and D are different, they both depend on x_1 , on the preference function $U(\cdot)$ and on the subjective probability distribution of e. In the absence of precise information about the nature of the objective function (1), this suggests that it is difficult to isolate the effect of R from the effect of D on current choices x_1 . In other words, it is possible that some type of risk behavior is attributed to timeless risk aversion when in fact it is due to the valuation of information (or vice versa). In order to solve this identification problem, good a priori information must exist on the nature of the objective function, the technology and the characterization of the uncertainty. However, good a priori information on preferences may be difficult to find. On this basis, it may be reasonable to focus our attention on how other factors (beside timeless risk preferences) can influence economic behavior under risk. In the next section, we briefly illustrate our argument in the context of a few examples.

3. Some examples

If the value of information $D(x_1, w)$ could be easily obtained, then expression (5) would provide a convenient basis for analyzing current economic decisions. Unfortunately, the value of information as defined in (2) can be rather complex, as the influence of some parametric change on D is not obvious in the general case. For example, although it may seem intuitive, it is not necessarily true that increasing uncertainty (as measured from a mean preserving spread) always increases the value of information [see Gould (1974), Hess (1982)]. This suggests that the discussion of the properties of the value of information and its implications can best proceed in the context of some specific examples.

⁴Expanding $U(w, x_2^*)$ around $\bar{x}_2(w, x_1, T)$ gives

 $U(w, x_2^*) = U(w, \bar{x}_2) + U_{x_2}(w, \bar{x}_2) \cdot [x_2^* - \bar{x}_2] + 1/2[x_2^* - \bar{x}_2]' U_{x_2x_2}(w, \bar{x}_2) \cdot [x_2^* - \bar{x}_2].$ Similarly, expanding $U(w + D, \bar{x}_2, \cdot)$ around D = 0 yields

 $U(w+D, \vec{x}_2(w+D), \cdot) = U(w, \vec{x}_2(w), \cdot) + \left[U_w + U_{x_2}(\partial \vec{x}_2/\partial w)\right]|_{w, \vec{x}_2} \cdot D.$

Using these two expressions, taking expectations, noting that $EU_{x_2}(\vec{x}_2) = 0$ (assuming an interior solution), and using (2) yields expression (7).

3.1. An irreversible decision

Consider the case of a fixed resource initially used in a particular way. At time t=1, the resource can be left in its initial state $(x_1=0)$ or transformed into some alternative use $(x_1=1)$. A similar choice is possible at time t=2, i.e. $x_2 = \{0 \text{ or } 1\}$. However, choosing $x_1=1$ is an irreversible decision in the sense that $x_1=1$ implies $x_2=0$, i.e. a loss of the option of choosing x_2 . Alternatively, choosing $x_1=0$ implies $x_2=\{0 \text{ or } 1\}$, leaving the option of choosing x_2 opened. Under temporal uncertainty, this problem can be formulated as

$$\max_{x_1} E \max_{x_2} \{ U(w, x_1, x_2, e) : x_t = \{ 0 \text{ or } 1 \}, \ t = 1, 2; \ x_1 + x_2 \le 1 \},$$
(8)

which is a special case of (1). Denote by $D(w, x_1)$ the conditional value of information in (8). Clearly D(w, 1) = 0: irreversibility implies no value of information for the second-period decision when $x_1 = 1$. Alternatively, the conditional value of information D(w, 0) is non-negative. In this case, D(w, 0) has also been called the quasi-option value [see Arrow and Fisher (1974), Henry (1974)].⁵

Using formulation (3), the first-period decision in (8) can be alternatively expressed as

$$\max_{x_1, x_2} \{ EU[w + D(w, x_1), x_1, x_2, e] : x_t = \{0 \text{ or } 1\}, t = 1, 2; x_1 + x_2 \le 1 \}.$$

It follows that the optimal choice is $x_1 = 1$ if

$$EU[w, 1, 0, e] \ge Max \{ EU(w + D(w, 0), 0, 1, e), EU(w + D(w, 0), 0, 0, e) \}.$$

Otherwise, choose $x_1 = 0$. This decision rule suggests that the conditional value of information D(w, 0) (the 'quasi-option' value) plays an important role in the evaluation of optimal decisions under irreversibility. For example, because future information can be of value only if $x_1 = 0$, the prospect of more information in the future can be shown to discourage the adoption of an irreversible decision $(x_1 = 1)$ in period 1 [see Epstein (1980)].

3.2. The firm under revenue uncertainty

The analysis of firm behavior under temporal price uncertainty has been presented by Epstein (1978) and Hartman (1976). Here, we focus on a

⁵Note that Arrow and Fisher (1974) assume that $U(w, x_1, x_2, e)$ is an additive function of x_1 and x_2 . The more general formulation presented here shows that the concept of quasi-option value is valid for any form of the objective function given irreversibility.

timeless risk-neutral case under revenue uncertainty. It can include both temporal price and production uncertainty. Consider a competitive firm with profit denoted by $\{p(e) \cdot y(x_1, x_2, e) - r'_1 x_1 - r'_2 x_2\}$ where p(e) is output price, $y(x_1, x_2, e)$ is the production function, the elements of the vector e denote output price uncertainty and/or production uncertainty, and $(r'_1 x_1 + r'_2 x_2)$ is the cost of production, r_t denoting the (discounted) price of inputs $x_t, t = 1, 2$. Assuming a linear utility function U(w) (i.e. timeless risk neutrality), the firm decides to make production choices that maximize expected profit under temporal uncertainty:

$$\max_{x_1 \ge 0} E\left\{\max_{x_2 \ge 0} \{p(e) \cdot y(x_1, x_2, e) - r'_2 x_2\} - r'_1 x_1\right\},$$
(9)

where $r'_1 x_1$ constitutes a fixed cost at time t=2. In the context of (9), the conditional value of information is

$$D(x_1) = E \max_{\substack{x_2 \ge 0}} \{ p(e) \cdot y(x_1, x_2, e) - r'_2 x_2 \}$$
$$- \max_{\substack{x_2 \ge 0}} E\{ p(e) \cdot y(x_1, x_2, e) - r'_2 x_2 \},$$

which measures the expected profit loss caused by choosing x_2 without observing e.

From (3), the first-period decision x_1 can be characterized by

$$\max \{ D(x_1) + \mathbb{E} \{ p(e) \cdot y(x_1, x_2, e) - r'_2 x_2 - r'_1 x_1 \} \}.$$
(10)
$$x_1 \ge 0 \\ x_2 \ge 0$$

Expression (10) indicates that the conditional value of information will influence the choice of the production decisions x_1 . For example, under competition, the first-order necessary conditions for an interior solution for x_1 are

$$\frac{\partial D}{\partial x_1} + \mathbf{E} \left[p(e) \frac{\partial y(x_1, \bar{x}_2, e)}{\partial x_1} \right] - r_1 = 0.$$
(11)

This shows that, at the optimum, the expected marginal value product with respect to x_1 [the second term in (11)] is greater than, equal to or less than the input price r_1 whenever the marginal value of information $\partial D/\partial x_1$ is

negative, zero or positive, respectively. In other words, the static optimality conditions (stating that expected marginal value product equals input price) would no longer hold when the marginal value of information is non-zero. In such a case, temporal uncertainty and information would affect the optimality of production decisions even under timeless risk neutrality.

In order to illustrate further the influence of temporal uncertainty on production decisions, consider the following specification for the production function:

$$y(x_1, x_2, e) = a(x_1, e) + x'_2 b(x_1, e) + \frac{1}{2x'_2} A(x_1) x_2,$$
(12)

where $A(x_1)$ is a negative definite matrix corresponding to the strict concavity of the production function in x_2 . The specification (12) is of interest because it allows an explicit evaluation of the value of information $D(x_1)$. In particular, if price uncertainty and production uncertainty are independently distributed, then the value of information for a timeless riskneutral firm takes the form

$$D(x_1) = -1/2E(p) \cdot \operatorname{Tr} \left\{ A(x_1)^{-1} \cdot \operatorname{Var} \left[b(x_1, e) \right] \right\}$$
$$-1/2r'_2 A(x_1)^{-1} r_2 \left\{ E\left(\frac{1}{p}\right) - \frac{1}{E(p)} \right\} \ge 0,$$
(13)

where $Var(\cdot)$ denotes the variance and $Tr(\cdot)$ is the trace. The first term in (13) represents the value of information associated with production uncertainty, while the second term is the value of information associated with output price uncertainty. Note that the first term is necessarily non-negative given the negative definiteness of $A(x_1)$. Similarly, the second term is non-negative since Jensen's inequality implies that E(1/p) > 1/E(p), 1/p being a convex function of p for p > 0.

It follows from (13) that the marginal value of information $\partial D/\partial x_1$ in (11) can be positive, zero, or negative depending on the nature of the production technology, i.e. depending on how $A(x_1)$ and the variance of $b(x_1, e)$ vary with x_1 . If eq. (12) is specified as a polynomial function, the absence of third or higher order terms involving interactions between x_1 , x_2 and e in the production function would correspond to a situation where the certainty equivalence principle holds [e.g. see Bertsekas (1976, p. 70)] with $\partial D/\partial x_1 = 0$. This illustrates the fact that second-order approximations of a function are

not flexible in modeling the impact of temporal uncertainty on economic decisions [see Epstein (1978)]. In other words, third-order (or higher order) interaction terms between x_1 , x_2 and e are needed in the production function (12) if one wants to avoid imposing strong restrictions on the way information influences production decisions in a dynamic context.

The above discussion illustrates that temporal revenue uncertainty can either stimulate or dampen production choice depending on the nature of the production technology. The results could be used in the modeling of many dynamic agricultural production processes. For example, the effectiveness of pest management strategies appears to depend heavily on the information available at the time of the decision. This suggests that the economics of production decisions may benefit from a detailed analysis of the role of information in the cost-benefit evaluation of alternative strategies. Some empirical evidence of the characterization of agricultural production technology and on the influence of information valuation is presented next in the context of sorghum yield response.

4. An application to yield response

Yield response is a dynamic process reflecting biological growth of the plant considered. The production process can be divided into time intervals or stages, state variables being used to characterize the dynamics of the process at various stages. Although the growth process of crops is a continuous one, we can categorize the most critical growth stages of the plant which correspond to the management activities taking place at that time.

We characterize the growth process by the following state equation:

$$u_{n+1} = f_n(u_n, x_n, e_n, T), \quad n = 0, 1, \dots, N-1,$$
(14)

where

 u_n = the state of the plant at a stage *n* (e.g., height, biomass), assumed to embody the effect of all inputs and random components in earlier stages;

- $x_n = a$ vector of 'controllable' decision variables affecting plant growth (e.g., weeding labor hours, kilograms of fertilizer);
- $e_n = a$ vector of 'uncontrollable' environmental factors reflecting the uncertainty resolved at stage *n* (e.g., rainfall);
- T = a variable representing the characteristics of technology (e.g., soil fertility measure);
- N = number of stages involved in the growth process.

If we take the initial state u_0 as given, it follows that

$$u_{n+1} = g_n(x_n, e_n; x_{n-1}, e_{n-1}; \ldots; x_1, e_1; x_0, e_0, T).$$

Assuming that yield, y, is a function of the state of the plant in its final stage, i.e. $y = k(u_N)$, we can express the yield function as

$$y = h(x_{N-1}, e_{N-1}; \dots; x_1, e_1; x_0, e_0; T).$$
(15)

which is a multi-stage production function. Expression (15) says that crop yield in the final stage depends on events of earlier stages and the given level of technology.

As discussed above, a relevant economic implication of the sequential nature of the decision-making process concerns the value of information. Here, we focus on a three-stage yield model (N=3) where the first stage is the planting stage, the second stage corresponds to weeding (or replanting in case of crop failure) and the third stage corresponds to harvesting.

In order to obtain analytical results concerning the value of information, the yield function (15) was specified according to the specification (12). Expression (12) can be interpreted as an 'extended' quadratic specification of the technological relationship between input and output. It appears to be a reasonable choice for two reasons. First, a quadratic function has often been found satisfactory in the analysis of yield response [e.g. Heady and Dillon (1961)]. Second, as discussed above, expression (12) includes third-order terms that are necessary if we do not want to impose strong restrictions on the way information influences production decisions.

More specifically, the following form for the yield response function was used:

$$y(x_1, x_2, e) = \alpha + \beta x_1 + x_1' B x_1 + x_2' [\gamma + Ce + G x_1 e] + 1/2x_2' A x_2,$$
(16)

where y is yield per hectare, x_n are decisions made at stage n, n=1,2, e represents the temporal uncertainty resolved between stage one and stage two, and A and B are symmetric matrices of parameters.

5. Empirical results

This section presents some empirical evidence concerning the influence of information on farm practices in Burkina Faso (West Africa). This is done first by estimating the yield response function (16) for sorghum. The implications of information valuation for production decisions are then interpreted in the context of expressions (10), (11) and (13) discussed above.

Data were taken from a survey of farm practices in Burkina Faso conducted from 1981 to 1983 by the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT). The data consist of white sorghum yield and related input information on 459 farm plots from the Yako region [Kristjanson (1987)]. The Yako region is representative of the Central Mossi Plateau where approximately 60 percent of Burkina Faso's population lives. Classified as the North Sundanian zone, it has a long-term average rainfall of 700–900 mm distributed over 4 to 5 months. Soils tend to be very shallow with low organic matter content. Sorghum is the dominant crop grown in this region. Very little animal traction is used.

Soil quality differs greatly and is strongly related to the position in the toposequence. Information on the type of soil (e.g. sandy, clayey), the position of the field in the toposequence (e.g. plateau, lowlands), and its proximity to the compound were used to define a proxy variable (soil type) to account for the predominant soil quality of each plot. The traditional variety of sorghum planted was identified as either short-cycle or long-cycle through the farmers' own identification of the cycle length of particular varieties. Other factors influencing yield include the number of labor hours spent on first weeding and second weeding, and the kilograms per hectare of added chemical fertilizer and/or manure. Table 1 summarizes some of the factors influencing sorghum yield.

Rainfall is considered to be the major risky variable influencing yields in the West African semi-arid tropics. Thus, the temporal uncertainty e in eq. (16) was taken to be the amount of rainfall during the first thirty days after planting. It was included as an explanatory variable in the model and is expected to have a strongly positive influence on yield.

Some critical interactions between explantory variables were identified and included in the model specification. The first-stage or x_1 decisions – what to plant, when to plant, whether to fertilize can be affected by the knowledge that information about rainfall will be gained before stage two decisions are made (i.e. replanting, weeding). In order to investigate such effects, selected third-order terms were included in the function (16) as interaction terms between the first-stage input decisions, rainfall e, and the second-stage decisions.⁶ The interaction terms, including the replanting decision, are of particular interest since the flexibility to replant may be an important element in risk management strategies in Burkina Faso. The manner in which informational effects may shift the input demand curves for the x_1 variables is discussed in table 2.

The multi-stage production function (16) was estimated by OLS. With the sequential decision model, the second-stage decision is assumed to be uncorrelated with the first-stage error term.⁷ This appears appropriate since

⁶Given the number of explanatory variables considered (see table 1), including all variables in the extended quadratic specification (16) generated a large number of parameters to estimate. In an attempt to reduce the number of parameters, the A and B matrices were assumed to be diagonal and some of the coefficients were assumed to be zero. This resulted in the model specification presented in table 3 below.

⁷If there were reason to believe a correlation did exist, a simultaneous equation method for estimating the model would be required.

Name of variable		Description of variable	Expected influence on yields and trade-offs involved
Soil type	X 11	Dummy variable; good soil = 1 bad soil = 0	Yields on clay soils are expected to be higher than those on sandy soils due to better water retention.
Date	<i>X</i> ₁₂	Date of planting in days (Mar. $1=0$)	Delayed planting may be expected to decrease yield given the short growing season.
Variety	Χ ₁ 3	Dummy variable; length of growing cycle of variety seeded; short-cycle = 1 long-cycle = 0	Long-cycle varieties may be higher yielding on average.
Tkgfert	X ₁₄	Total kilograms per hectare of chemical fertilizer applied	Added chemical fertilizer is expected to have a positive influence on yield (with sufficient rainfall).
Tkgman	X ₁₅	Total kilograms per hectare of manure applied.	Added manure should have a positive effect on yield.
Hrswd1	X 21	Hours of labor spent on first weeding task	Increased labor hours should increase yields.
Hrswd2	X 22	Hours of labor second weeding	
Pcarrep1	X 23	Percentage of the area of the plot replanted	Replanting increases plant density and should have a positive effect on yield.
Rainfall	е	Total millimeters in first 30 days after planting	The amount of rainfall in the first stages of growth is expected to have a strongly positive effect on yield.

Table 1 Explanatory variables

rainfall (the major risk variable) is included as an explanatory variable (rather than modeled as part of the unexplained error term).

Based on 459 plot level observations, the results of the fitted functions for sorghum are presented in table $3.^8$ The *F*-value is significant at the 1 percent level (leading to the rejection of the null hypothesis that all coefficients are zero). The R^2 is 0.44, with 12 out of 16 variables having significant coefficients at the 10 percent level.

The soil type variable is strongly significant indicating the responsiveness of sorghum to better soil types (i.e., a clay soil, with better water holding capacity, gives higher production). The date of planting and the use of short-

⁸Attempts to include fertilizer and manure as quadratic terms $(X_{14}^2 \text{ and } X_{15}^2)$ in the model gave corresponding coefficients that were positive and not significantly different from zero. This suggests that our sample may have too few observations involving high enough fertilizer and manure use to allow a precise estimation of the region of the production function where marginal products are declining. The results presented below correspond to the model where X_{14}^2 and X_{15}^2 have been omitted.

x ₁ decision	Hypothesized influence of x_1 on D	Explanation
Timing of planting (X_{12})	Negative	The earlier planting is done, the more flexible a position is adopted and the higher is the value of information.
Application of chemical fertilizer and manure (X_{14})	Either positive or negative	
The use of short-cycle varieties (X ₁₃)	D higher for short-cycle varieties than for long-cycle varieties	Varieties with shorter cycles offer more options in terms of timing of planting and the opportunity to replant and thus new information has a higher value to the farmer than with long-cycle varieties.

Table 2The influence of selected variables on the value of information $D(x_1)$.

cycle traditional varieties have a negative influence on sorghum yield. Both the amount of time spent on the first weeding and the second weeding are highly significant, indicating the importance of timely weeding of sorghum to achieve better yields. Chemical fertilizer is found to have a positive effect on yield through its interaction with rainfall and replanting. Finally, manure use is significant but only at the 10 percent level. It may be that our soil type variable is picking up the influence of manure, since compound land (which receives most of the household manure) was included in the characterization of 'good soil'.

Squared variables that were included have the expected signs. Interaction effects (1) between variety, rainfall and percentage area replanted, (2) between fertilizer use, rainfall and percentage area replanted, and (3) between date of planting, rainfall and first weeding hours, were all highly significant.

Marginal physical products and elasticities of production are given in table 4. The productivity of manure on sorghum fields appears to be very low. This may be due to the fact that the residual effect of manure is not captured in the model. Also, as mentioned earlier, it may be that our soil type variable is in fact capturing the benefits of manure on yield.

The elasticity of production with respect to rainfall is positive with a value of 0.41 for long-cycle varieties and 0.73 for short-cycle varieties. The response in yield to an increase in the area replanted is positive. This can be expected if replanting increased yields due to increased plant density. These results appear reasonable, thus suggesting that the extended quadratic specification (16) provides a possibly reasonable characterization of the underlying technology.

An hypothesis test of the significance of the included three-way interaction terms (the 'information variables') was performed, the null hypothesis being

	Tiela function it	or sorghum (kg/ma).
Intercept		258.57***
Soil type	X ₁₁	110.2***
-		(33.7)
Date	X ₁₂	-1.5***
		(0.44)
Variety	X ₁₃	-70.3**
T 1 C	V	(35.1)
Ikgtert	X 14	0.37
T 1	v	(0.00)
ткутап	X 15	(0.01)
Hesudi	v	0.01)
mowur	A 21	(0.19)
Hrswd2	X	0.92***
11130002	1 22	(0.19)
Pearrent	X.,	-0.79
	23	(0.51)
	X_{21}^{2}	- 0.00004
	21	(0.00001)
	X_{22}^{2}	0.0004***
		(0.0001)
	X_{23}^2	- 0.000007
		(0.0007)
	X ₂₃ e	4.2***
		(1.05)
	X ₁₃ e X ₂₃	3.2***
		(0.98)
	$X_{14} e X_{23}$	0.04***
		(0.01)
	X ₁₂ e X ₂₁	-0.00001***
		(0.00006)

Table 3Yield function for sorghum (kg/ha).*

 ${}^{a}n = 459$; R^{2} value: 0.4440; F value: 24.17; Standard errors are in parentheses. Significance levels are: *0.10; **0.05; ***0.01.

G=0, where G is the vector of coefficients of the third-order terms in (16). With an F value of 5.7, the null hypothesis that the three-way interaction terms are not significantly different than zero is rejected at the 1 percent significance level. This illustrates the fact that information considerations can influence risk behavior even under a utility function linear in profit. In the case, when the timeless risk premium is zero, the temporal risk premium is the conditional value of information [see eq. (5)] which affects the first-period decisions whenever x_1 affects the value of information D. Under a linear utility function U(w), from (13), the production function (16) implies that the value of information associated with production uncertainty is

Variable	MPP	E,			
Date of planting (X_{12})	-2.17	-0.17			
Fertilizer (X_{14})	2.5	0.05			
Manure (X_{15})	0.018	0.009			
First weeding (X_{21})	0.6	0.3			
Second weeding (X_{22})	0.75	0.3			
Percentage area replanted (X_{23})					
Long-cycle varieties	72.5	0.05			
Short-cycle varieties	562.1	0.36			
Rainfall (e)					
Long-cycle varieties	1.45	0.41			
Short-cycle varieties	2.57	0.73			

Table 4 Marginal physical products and elasticities of production for white sorghum.

^aMPP = marginal physical product = $\hat{c}y/\hat{c}x$, E_p = production elasticity = $(\hat{c}y/\hat{c}x) \cdot (x/y)$, evaluated at mean input levels.

$-1/2E(p) \operatorname{Tr} \{A^{-1} \cdot [C + Gx_1] \operatorname{Var} (e) [C + Gx_1]'\}.$

Under temporal production uncertainty, the significance of the three-way interaction terms (see table 3) indicates that information would have a significant influence on the following x_1 decisions: the date of planting, fertilizer use, and the choice of varieties. Here, it is of interest to discuss briefly the relationship between D and x_1 . Given the model specification (16), finding $\partial D/\partial x_1 < 0$ would imply that flexibility considerations can shift down the input demand curve for x_1 , resulting in a lower optimum level of the input x_1 (compared to the case where information is not accounted for). The opposite would be true if $\partial D/\partial x_1 > 0$. Thus the sign of the marginal valuation of information will influence the direction in the shift of the demand curve.

As an example, the model specification reported in table 3 implies that, under timeless risk neutrality, the marginal value of information with respect to the date of planting would be negative. This suggests that the optimum date of planting as determined by an open-loop model would be later in the season than the optimum date obtained from a sequential model. The later a farmer plants, the less chance he has to have time to adjust his decisions and the lower the value of new information. Thus, our results indicate that, by planting earlier, a more flexible position is adopted, which allows a better use of rainfall information in production decisions. This flexibility may help explain why many farmers in the Sahel tend to plant very early in the rainy season. This incentive to plant early should be taken into consideration in making recommendations concerning cultural practices in the Sahel.

Another example is the influence of varietal choice on the value of information $D(x_1)$. Comparing short-cycle and long-cycle varieties under timeless risk neutrality, the model reported in table 3 implies that D is higher

for short-cycle than for long-cycle varieties. This result can be interpreted as follows. Seed varieties that have a shorter growing cycle can be planted over a wider period than long-cycle varieties. They would therefore offer more options in terms of timing of planting and the opportunity to replant. This would increase the value of information, as suggested by our empirical results. In other words, the flexibility of production plans may give an incentive to plant short-cycle (rather than long-cycle) varieties even if, ceteris paribus, the short-cycle yields are a little lower than the long-cycle yields. Given the perceived increase in rainfall uncertainty in the Sahel, this incentive may help explain the observed switch made by Burkino Faso's farmers of the past 20 years from long-cycle to short-cycle sorghum varieties.

These results indicate the potential influence of information valuation on risk preferences and economic behavior. It shows that the temporal resolution of uncertainty can generate risk behavior (even in the case of timeless risk neutrality). However, our empirical results should be interpreted with caution. For example, we limited our analysis to a two-period model. Extending the investigation to more than two periods appears desirable.⁹ Note that it would imply that our timeless utility function $U(\cdot)$ in section 2 would, in fact, be an induced preference function from the third, fourth, etc. periods. In other words, our timeless risk premium $R(\cdot)$ in (5) would in fact be a temporal risk premium that would depend on the valuation of the information becoming available in the third, fourth, etc. periods. This suggests the need for further research on the role of information valuation in risk behavior.

6. Concluding remarks

This paper has developed a method to investigate the role of temporal uncertainty and information issues in economic decisions. It suggests that the nature of the economic environment (e.g. the production technology) can imply that the choice functions can be affected by uncertainty (e.g. as measured by a variance) even under a linear utility function (i.e. timeless risk neutrality). This raises the question of the identification of timeless risk aversion in economic behavior. For example, it is possible that some type of risk behavior is, in fact, due to the valuation of information. This has been illustrated by a few examples and by an empirical application to yield response analysis.

This paper has presented some evidence on the influence of technology and temporal uncertainty on risk behavior. It has been emphasized that, in

⁹Another possible extension would be to make the model more realistic by increasing the number of decision variables at each time period. This case has been discussed by Machina (1984).

general, objective functions that provide second-order approximations (commonly found in empirical research) are *not* flexible in the investigation of information issues. The empirical evidence presented in the context of sorghum yield response under temporal rainfall uncertainty indicates that the valuation of information can be expected to influence production decisions. The interaction terms playing a role in information valuation were found to be significant, leading to the conclusion that temporal rainfall uncertainty and flexibility issues can play an important role in agricultural production decisions in Burkina Faso.

By stressing the importance of technology and information valuation in risk behavior, this paper has proposed a new direction of research on risk analysis. This new direction should be of interest to the extent that temporal uncertainty appears to be the rule rather than the exception in the real world. Also, in the case where the institutional environment of decision makers can influence information valuation, this would allow policy instruments to influence directly risk behavior. For example, noting that uncertainty seems to play an important role in the process of agricultural development [e.g. Roumasset et al. (1978)]. our approach may provide new insights in the design and evaluation of development policy. It is hoped that this research will stimulate additional empirical work which can help refine our understanding of the role of information in economic analysis and policy.

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