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A STATISTICAL PROCEDURE FOR TESTING THE HOST PLANT PERFERENCE UNDER CHOICE AND NO CHOICE SITUATION *

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By

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A statistical procedure has been developed to test the preference of a subject towards one of two options which are used singly and combined. The test statistics are based on generalised least square (GLS) estimates of the parameters in a linear model with correlated erros and on the likelihood ratio test. The method is used to test the preference of the egg parastie (**Trichogramma brasiliensis**) for sorghum to pigeonpea in choice and no choice tests.

INTRODUCTION

Sometimes an investigator may be interested in comparing the preference for a subject towards one of the two options when given a choice and the extent of such a response being expressed in the absence of choice.

In behaviour studies with insects, the preference for a host crop is often assessed by a choice test (keeping the contrasting hosts together) followed by a no choice test, (keeping the hosts alone with no scope for response by comparison) so to ascertain the consistency of any preference observed. This situation involves comparing the preference of the insect towards one of the two options, say P or S, using the information on P and S when they are adjacent, which enables the direct choice between P and S, and also when P is alone and S is alone. Laboratory tests on an insect parasitoid, in relation to preference between two target crops which had the same host insects constituted the base for a statical modelling developed to interpret the data on the above lines, as will be described below. Statistical

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Species of *Trichogramma* (Trichogrammatide Hymenoptera) are common parasitoids attacking the eggs of several insect pests, mainly Lepidoptera. *Heliothis armigera* (Hb) (Noctuidae, Lepidoptera) is a pest of a number of crops in the semi-arid tropics (SAT) including sorghum (*Sorghum bicolor*) and pigeonpea (*Cajanus cajan*). At ICRISAT, Natural parasitism of *H. armigera* eggs by *Trichogramma* (mainly *T. chilonis*) had been observed to be substantial on sorghum, but negligible on pigeonpea (Bhatnagar *et al.*, 1982). A series of tests were therefore initiated to screen exotic species of *Trichogramma* for their host plant preference to see if they preferred pigeonpea better than the local *T. chilonis*. In a laboratory tests, an exotic species, *T. bras-iliensis* was studied for its preference to its host eggs on sorghum versus pigeonpea employing choice and no choice tests. Flowering terminals of the two crops were kept as fresh bouquets, eggs of *H. armigera* fixed on them and the two crops were kept in cages, both together and separately. A flux of adult *T. brasiliensis* was released into each cage and the parasitism (% eggs parasitised) was scored after one week. Five replications were kept.

A correlation structure on the errors of linear models to account for the trend of response in preference for one crop, when both the crops are kept together as well as separately, is introduced in section 2. For a hypothetical situation when this correlation is known, the generalised least squares (GLS) method of analysis may be used (Aitken, 1935; Rao, 1973). The results based on GLS method are explicitly furnished in section 3. Since, in practice, correlation between errors is unknown, a statistic has been derived, using the likelihood ratio test (Kendall and Stuàrt, 1979) for comparing the preference of the parasite (*T. brasiliensis*)) for one crop (say, sorghum). The method of analysis for percentages based on unequal number of total observations is presented in section 5 and illustrated in section 6 with the sorghum-pigeonpea data.

Though the notations used are for this experiment, the procedure can be usefully applied to process data from similar investigations on preference for targets.

MODEL FORMULATION

Let Y_{1i} and Y_{2i} refer to the observations on the percentage of parasitized eggs on the pigeonpea and sorghum terminals in separate cages and Y_{3i}, and Y_{4i}, the observations on pigeonpea and sorghum terminals when in the same cage of ith replicate (i = 1, 2, . . . , n, being the number of replications. A set of observations have been shown in Table I. We can represent this by

i-th replicate (i = 1, 2, . . . , n)

Separate cages		Same cage	
Pigeonpea	Sorghum	Pigeonpea	Sorghum
Y_{1i}	Y_{2i}	Y_{3i}	Y_{4i}

Observation

Further, let $\alpha(p \to p)$ be the attraction effect on the parasite for pigeonpea which parasitise the eggs on pigeonpea and $\alpha(p \to s)$ the attraction effect for pigeonpea when they parasitise eggs on sorghum, similarly $\alpha(s \to p)$ and $\alpha(s \to s)$ refer to the attraction effects of the parasite on sorghum parasitizing eggs on pigeonpea and on sorghum respectively. We can represent the Y-observations in terms of α's in the following linear model :

$$Y_{1i} = \alpha(p \to p) + \varepsilon_{1i} \qquad i = 1, \ldots, n$$

$$Y_{2i} = \alpha(s \to s) + \varepsilon_{2i}$$

$$Y_{3i} = \alpha(p \to p) + \alpha(s \to p) - \alpha(p \to s) + \varepsilon_{3i}$$

$$Y_{4i} = \alpha(s \to s) + \alpha(p \to s) - \alpha(s \to p) + \varepsilon_{4i} \tag{2.1}$$

where the errors are assumed normally distributed with means $E(\varepsilon_{1i}) = E(\varepsilon_{2i}) = E(\varepsilon_{3i}) = E(\varepsilon_{4i}) = 0$ and variance-covariance values

$$V(\varepsilon_{1i}) = V(\varepsilon_{2i}) = V(\varepsilon_{3i}) = V(\varepsilon_{4i}) = \sigma^2$$

$$\text{Cov}(\varepsilon_{3i}\ \varepsilon_{4i}) = \rho\, \sigma^2, \text{ say } i = 1, \ldots, n$$

and covariances between all other ε s zero. Define

$$Y'_1 = (Y_{11} \ldots Y_{1n});\ Y'_2 = (Y_{21} \ldots Y_{2n})\ ;\ Y'_3 = (Y_{31} \ldots Y_{3n})\ ;$$

$$Y'_4 = (Y_{41} \ldots Y_{4n})\ ;$$

and $Y' = (Y'_1, Y'_2, Y'_3, Y'_4)$;

$$\tau_p = \alpha(p \to p)\ ;\ \tau_s = \alpha(s \to s),\ \delta = \alpha(s \to p) - \alpha(p \to s)$$

The model (2.1) can be written as

$$Y = X\beta + \epsilon$$

where $\beta = (\tau_p, \tau_s, \delta)$;

$X = Z \circledast J_n$;

$E(\varepsilon) = 0$;

$D(\varepsilon) = E(\varepsilon\varepsilon') = \sigma^2 C \circledast I_n$; (2.2)

$$Z = \left| \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{matrix} \right.$$

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \rho \\ 0 & 0 & \rho & 1 \end{matrix}$$

J_n is a n-component column vector of unities and I_n, the identity matrix of order n_o $\circledast$ represents the Kronecker product given by (for $A = (a_{ij})$ sxt, $B = (b_{ij})$ uxv).

$$A \circledast B = ((a_{ij} B)) \text{ su x tv}$$

where, c $B = ((cb_{ij}))$;

E (.) stands for the expectation and D (.) for the variance-covariance matrix.

Consider the coefficient $\delta = \alpha$ (s→p) —α (p→s) which is a measure of the additional preference of the parasite for the host eggs on pigeonpea. When both species are equally preferable δ is zero. Thus a test for preference is to test the significance of δ from zero. In the next two sections we develop the tests for the null hypothesis Ho : $\delta = 0$.

TEST OF NULL HYPOTHESIS WHEN ρ IS KNOWN

If the correlation coefficient ρ is known the best linear unbiased estimators of the elements of β are the generalized least squares (GLS) estimates (see Aitken, 1935 and Rao, 1973, pages 220-230).

The generalized least squares estimator of β is obtained by minimizing

$$\Phi = (Y - X\beta)' (D(\varepsilon))^{-1} (Y - X\beta) \tag{3.1}$$

with resp:ct to β_o If we write $V = C \circledast I_n$

,

then the GLS estimator $\widehat{\beta}$ to β

is $$\widehat{\beta} = (X'V^{-1}X)^{-1} X V^{-1} Y \tag{3.2}$$

and the dispresion matrix of $\widehat{\beta}$ is

$$D(\widehat{\beta}) = \sigma^2 (X'V^{-1}X)^{-1} \tag{3.3}$$

The expression for $\widehat{\beta} = (\widehat{\tau}_p, \widehat{\tau}_s, \widehat{\delta})'$ simplifies to the following explicit forms

$$\widehat{\tau}_p = ((3+2\rho) Y_1 - Y_2 + Y_3 + Y_4) / (2(2+\rho))$$

$$\widehat{\tau}_s = (-Y_1 + (3+2\rho) Y_2 + Y_3 + Y_4) / (2(2+\rho))$$

$$\widehat{\delta} = ((Y_3 - Y_1) - (Y_4 - Y_2)) / 2$$

where $Y_u = \sum_{i=1}^{n} Y_{ui} / n$ for u = 1, 2, 3, 4

The variances and covariances between $\widehat{\tau}_p$, $\widehat{\tau}_s$ and $\widehat{\delta}$ obtained from (3.3) are reproduced below :

$V(\widehat{\tau}_p) = V(\widehat{\tau}_s) = \sigma^2 (3+2\ ) / (2n(2+\rho))$, $Cov(\widehat{\tau}_p, \widehat{\tau}_s) = -\sigma^2, (2n(2+\rho))$

$V(\widehat{\delta}) = \sigma^2 (2-\rho) / (2n)$, $-Cov(\widehat{\tau}_p, \widehat{\delta}) = Cov(\widehat{\zeta}_s, \widehat{\delta}) = \sigma^2 / (2n)$

An unbiased estimator of σ^2 is obtained by computing error sum of squares

$$SSE = Y^1 V^{-1} Y \widehat{\beta}' \underset{\sim}{x}' V^{-1} Y$$

Denote by

$$SS_{uv} = Y'_u Y_v \quad u, v = 1, 2, 3, 4$$

the various sum of squares and products for sorghum and pigeonpea vectors. It can be seen that

$$SSE = SS_{11} + SS_{22} + (SS_{33} + SS_{44} - 2\rho\, SS_{34}) / (1-\rho^2)$$

$$-n\ \{\hat{\tau}_p\ ((1-\rho^2)\,Y_1 + Y_3 - \rho Y_4) + \hat{\tau}_s\ ((1-\rho^2)\,Y_2 - \rho Y_3 + Y_4)$$

$$+\ (1+\rho)(Y_3 - Y_4)\hat{\delta}\ \}$$

Thus we can estimate σ^2 unbiasedly by the mean sum of squares (MSE)

$$MSE = SSE\ /\ (4n-3).$$

A test for the hypothesis Ho ; $\delta = 0$ is based on computing the statistics

$$Q_1 = \hat{\delta}\ /\ ((2-\rho)\ MSE\ /\ (2n))^{\frac{1}{2}}$$

and compa ing (computed) Q_1 against a (tabulated) t —value with 4n-3 d. f.

TEST FOR THE NULL HYPOTHESIS WHEN ρ IS UNKNOWN

A more realistic situation points towards an unknown value of ρ in model (2.2). In the following we consider the test Ho: $\delta = 0$ based on the maximum likelihood ratio. Under the assumption of normality of errors ε in (2.2), we have the likelihood for β, σ^2 and ρ based on the observations used in the previous section as

$$L(\beta,\ \sigma^2,\ \rho/Y) = (1/(2\pi)^{2n}\sigma^{4n}\ |V|)\exp(-(Y-X\beta)'\ V^{-1}(Y-X\beta))/(2\sigma^2)) \qquad (4.1)$$

where the determinant $|V| = (1-\rho^2)^n$. The likelihood ratio test is based on the ratio of the maximum value of the likelihood function (4.1) under the restriction of the null hypothesis to its maximum value obtained ignoring this restriction.

The maximum likelihood estimator (m.l.e.) of β, σ^2 and ρ are obtained by maximizing L or ln $L = L_1$.

$$= Co - 2n\ln\ \sigma^2 - (n/2)\ \ln(1-\rho^2) - (Y-X\beta)\ V^{-1}\ (Y-X\beta)\ /2\sigma^2. \qquad (4.2)$$

where $Co = -2n\ln(2\pi)$. The maximizing equations are given in Appendix I.

Denoting m.l e. of β, σ^2, ρ by $\hat{\beta}_m$, $\hat{\sigma}^2_m$ and $\hat{\rho}_m$ respectively the equations we have

$$\hat{\beta}_m = (X'V^{-1}X)^{-1}\ X'V^{-1}\ Y \qquad (4.3)$$

$$\hat{\sigma}^2_m = (Y - X\hat{\beta}_m)' V^{-1} (Y - X\hat{\beta}_m) / 4n \quad (4.4)$$

$$g(\rho) = -\sigma^2 n\rho(1-\rho^2) - (1-\rho^2) SS_{44} + \rho(SS_{33} + SS_{44} - 2\rho SS_{34}) + 2n$$

$$\{\rho(\tau_p^2 + \tau_s^2) + \delta^2 (1+\rho)^2 - \tau_p \tau_s (1+\rho^2) + \delta(\tau_p - \tau_s)(1+\rho)^2\}$$

$$+2\rho[\{(1+2\rho-\rho^2)Y_4 - 2Y_3\}\tau_p + \{(1+2\rho-\rho^2)Y_3 - 2Y_4\}\tau_s]n$$

$$-2(1+\rho)^2 n\delta(Y_3 - Y_4) = 0 \quad (4.5)$$

The equations (4.3), (4.4) and (4.5) can be solved iteratively using Newton-Raphson method (Stark, 1970, pages 85-90) given in Appendix II.

The asymptotic variance-covariance matrix V_a of $\hat{\rho}_m$, $\hat{\sigma}^2_m$ and $\hat{\beta}_m$ can be obtained as

$$V_a = \begin{pmatrix} 2(1-\rho^2)^2/(4+\rho^2)n & \rho\sigma^2(1-\rho^2)/(4+\rho^2)n & 0 \\ & \sigma^4(2+\rho^2)/(4+\rho^2)n & 0 \\ \text{Symm.} & & \sigma^2(X'V^{-1}X)^{-1} \end{pmatrix}$$

following the steps given in Appendix III. Here symm. indicates the symmetry of the matrix.

In order to consider maximum likelihood ratio test for Ho: $\delta = 0$, we obtain the maximum values of L (or L_1) under Ho and ignoring Ho. The maximum value of L_1 ignoring Ho is obtained by substituting m.l.e. 's ($\hat{\rho}_m$, $\hat{\sigma}^2_m$, $\hat{\beta}_m$) in (4.2) which turns out to be

$$L_1 = -2n\ln\hat{\sigma}^2_m - (n/2)\ln(1-\hat{\rho}^2_m) - 2n$$

Under Ho, the estimates $\hat{\hat{\beta}} = (\hat{\hat{\tau}}_p, \hat{\hat{\tau}}_s, 0)'$, $\hat{\rho}$ and $\hat{\hat{\sigma}}^2$ of β, ρ and σ^2 of can be obtained from the following equation (computed iteratively) derived along the lines of section 3 and expressions (4.4) and (4.5).

$$\hat{\hat{\tau}}_p = \{ (2-\rho^2) Y_1 + \rho Y_2 + 2 Y_3 - \rho Y_4 \} / (1-\rho^2)$$

$$\hat{\hat{\tau}}_s = \{ \rho Y_1 + (2-\rho^2) Y_2 - \rho Y_3 + 2 Y_4 \} / (1-\rho^2)$$

$$\hat{\hat{\sigma}}^2 = (Y - X\hat{\hat{\beta}})' V^{-1} (Y - X\hat{\hat{\beta}}) / 4n$$

Equation for $\hat{\hat{\rho}}$:

$$g_0(\rho) = -\hat{\hat{\sigma}}^2 n\rho (1-\rho^2) - (1-\rho^2) SS_{34} + \rho (SS_{33} + SS_{44} - 2\rho SS_{34})$$

$$+ 2n \{ \rho (\hat{\hat{\tau}}_p^2 + \hat{\hat{\tau}}_s^2) - 2(1+\rho^2) \hat{\hat{\tau}}_p \hat{\hat{\tau}}_s \}$$

$$+ 2\rho \{ ((1+2\rho-\rho^2) Y_4 - 2Y_3) \hat{\hat{\tau}}_p + ((1+2\rho-\rho^2) Y_3 - 2Y_4) \hat{\hat{\tau}}_s \} n$$

$$= 0.$$

The maximum value of L_1, under Ho is

$$\hat{\hat{L}}_1 = -2n \ln \hat{\hat{\sigma}}^2 - (n/2) \ln (1-\hat{\hat{\rho}}^2) - 2n$$

The test for Ho, is based on the statistic Q_2

$$= -2[\hat{\hat{L}}_1 - \hat{L}_1] = 2[\hat{L}_1 - \hat{\hat{L}}]$$

$$= n[4 \ln (\hat{\hat{\sigma}}^2 / \hat{\sigma}_m^2) + \ln \{ (1-\hat{\hat{\rho}}^2) / (1-\hat{\rho}_m^2) \}]$$

The asymptotic distribution of Q_2 is chisquare on one degree of freedom. Thus this test will be effective for large sample situations.

TEST USING TRANSFORMED DATA

The procedure in the previous section is suitable when the response measured is from a normal distribution with constant variance. For counts or proportions a suitable transformation is required before the analysis can be done. When proportions are based on unequal numbers a weighted analysis is required.

Usually in a binomial proportion situation one of the two transformation are made (see Rao, 1973, page 427).

(i) *Arcsine transformation.* If m numbers of one type are observed out of n, from a binomial distribution B (n, p), then the transformed variable y is given by

$$y = \sin^{-1}\sqrt{(m/n)} \quad (5.1)$$

with the variance

$$\sigma^2 = 1/4n$$

which differs for estimated proportions based on different numbers.

(ii) *Logit transformation.* The logit transformation is given by

$$y = \tfrac{1}{2}\log\left(m/(n-m)\right) \text{ with the variance} \quad (5.2)$$

$$\sigma^2 = p\,(1-p)/n$$

which depends on population proportion and total number also. An estimate of this variance σ^2 is $m\,(n-m)/n^3$. Thus our model on the transformed variable is

$$Y_{1i} = \zeta_1 + \varepsilon'_{1i}$$

$$Y_{2i} = \zeta_2 + \varepsilon'_{2i}$$

$$Y_{3i} = \zeta_1 + \delta + \varepsilon'_{3i}$$

$$Y_{4i} = \zeta_2 - \delta + \varepsilon'_{4i} \quad (5.3)$$

Where Y_{1i} and Y_{2i} are respectively the responses on pigeonpea and sorghum

under separate cages and Y_{3i} and Y_{4i} on them under same cage in the ith replicate: ζ_1, ζ_2 are the effect on pigeonpea and sorghum and δ is the difference when kept in the same cage. Furthermore, ε's have zero expectations and variances which either are known (as in the case of the arcsine transformation) or can be estimated (as in the case of the logit transformation). Let us define weights as the inverse of the variances for ϵ's and denote by

$$W_{ji} = 1/v\,(\varepsilon'_{ji}) \qquad j = 1, 2, 3, 4,\ i = 1 \ldots\ldots N. \quad (5.4)$$

The log-likelihood function from (5.3) is

$$L_2 = \text{constant} - (N/2)\ln(1-\rho^2) - (\tfrac{1}{2})\Sigma W_{1i}(Y_{1i}-\zeta_1)^2$$

$$-(\tfrac{1}{2})\Sigma W_{2i}(Y_{2i}-\zeta_2)^2-(\tfrac{1}{2})Q/(1-\rho^2) \qquad (5.5)$$

where

$$Q=\Sigma W_{3i}(Y_{3i}-\zeta_1-\delta)^2-2\rho\Sigma\sqrt{W_{3i}W_{4i}}(Y_{3i}-\zeta_1-\sigma)(Y_{4i}-\zeta_{2}+\delta)+$$

$$\Sigma W_{4i}(Y_{4i}-\zeta_2+\sigma)^2$$

The maximum likelihood equations for estimating ζ_1 ζ_2 and δ are obtained from

$$M_i\beta = C$$

where

$$M_1 = \begin{vmatrix} m_{11} & m_{12} & m_{13} \\ & m_{22} & m_{23} \\ \text{Symm.} & & m_{33} \end{vmatrix}, \quad C = \begin{vmatrix} C_1 \\ C_2 \\ C_3 \end{vmatrix}$$

$$\beta = (\zeta_1, \zeta_2, \delta)'$$

$$m_{11} = (1-\rho^2)\Sigma W_{1i} + \Sigma W_{3i}$$

$$m_{12} = -\rho\Sigma\sqrt{W_{3i}W_{4i}}$$

$$m_{13} = \Sigma W_{3i} + \rho\Sigma\sqrt{W_{3i}W_{4i}}$$

$$m_{22} = (1-\rho^2)\Sigma W_{2i} + \Sigma W_{4i}$$

$$m_{23} = -(\Sigma W_{4i} + \rho\Sigma\sqrt{W_{3i}W_{4i}})$$

$$m_{33} = \Sigma W_{3i} + \Sigma W_{4i} + 2\rho\Sigma\sqrt{W_{3i}W_{4i}}$$

$$C_1 = (1-\rho^2)\Sigma W_{1i} Y_{1i} + \Sigma W_{3i} Y_{3i} - \rho\Sigma\sqrt{W_{3i} W_{4i}} Y_{4i}$$

$$C_2 = (1-\rho^2)\Sigma W_{2i} Y_{2i} + \Sigma W_{4i} Y_{4i} - \rho\Sigma\sqrt{W_{3i} W_{4i}} Y_{3i}$$

$$C_3 = \Sigma W_{3i} Y_{3i} - \Sigma W_{4i} Y_{4i} - \Sigma\sqrt{W_{3i} W_{4i}} (Y_{4i} - Y_{3i})$$

The equation for estimating ρ is

$$\rho N(1-\rho^2) - \rho Q(\rho) + (1-\rho^2)\Sigma\sqrt{W_{3i} W_{4i}}(Y_{3i} - \zeta_1 - \delta)(Y_{4i} - \zeta_2 + \delta) = 0$$

which can be solved iteratively. The asymptotic variance-covariance matrix of $\hat{\beta}$ is inverse of M_1 and variance of $\hat{\rho}$ is $(1-\rho^2)^2 / (N(1+\rho^2))$

To test the hypothesis : $\delta = 0$ the procedure in section 4 can be used.

The test statistics $Q_3 = -2 \ln\lambda$ in this case simplifies to :

$$Q_3 = N \ln((1-\hat{\rho}_0^2)/(1-\hat{\rho}^2)) + \Sigma W_{1i}(Y_{1i} - \hat{\zeta}_1^0)^2 - \Sigma W_{2i}(Y_{2i} - \hat{\zeta}_2^0)^2 -$$

$$\Sigma W_{1i}(Y_{1i} - \hat{\zeta}_1)^2 - \Sigma W_{2i}(Y_{2i} - \hat{\zeta}_2)^2 + Q_0/(1-\hat{\rho}_0^2) - Q/(1-\hat{\rho}^2)$$

where $\hat{\rho}_0$, $\hat{\zeta}_1^0$, $\hat{\zeta}_2^0$ are the maximum likelihood estimates of $\hat{\rho}$, $\hat{\zeta}_1$, $\hat{\zeta}_2$ under null hypothesis : $\delta = 0$; Q_0 is the value of expression Q defined above while $\hat{\rho}$, $\hat{\zeta}_1$, $\hat{\zeta}_2$ and δ in Q are maximum likelihood estimates of ρ, ζ_1, ζ_2 and δ without any hypotheses on parameters.

ILLUSTRATION

The number of eggs parasitized by *T. brasiliensis* (m) and total number of eggskept on the sorghum and pigeonpea (n) for five replicates are shown in table I.

Table 1—The total number of eggs (n) and those parasitized (m) on pigeonpea and sorghum

Separate cages				Same cage			
Pigeonpea		Sorghum		Pigeonpea		Sorghum	
m	n	m	n	m	n	m	n
37	91	39	80	19	82	15	60
17	89	0	53	0	80	22	107
36	90	42	106	0	42	19	88
6	82	41	92	0	70	23	64
0	78	82	128	0	49	14	66

The analysis given in section 5 was done using the arcsine transformation with weights 4n. We found the following estimates of the parameters of the model (5.3)

$$\hat{\zeta}_1 = 0.3411$$

$$\hat{\zeta}_2 = 0.6088$$

$$\hat{\delta} = -.0518$$

$$\hat{\rho} = 0.3198$$

with estimated variances

$$\hat{V}(\hat{\zeta}_1) = .00052 \qquad \hat{Cov}(\hat{\zeta}_1, \hat{\zeta}_2) = .00012$$

$$\hat{V}(\hat{\zeta}_2) = .00050 \qquad \hat{Cov}(\hat{\zeta}_1, \hat{\delta}) = -.00031$$

$$\hat{V}(\hat{\delta}) = .00058 \qquad \hat{Cov}(\hat{\zeta}_2, \hat{\delta}) = .00032$$

$$\hat{V}(\hat{\rho}) = .14623$$

The computed value of the likelihood ratio test statistic Q_3 is 5.04 which is significant at the five percent probability level. These data indicate that *T. brasilensis* prefers sorghum to pigeonpea.

By this study, it became evident that the exotic species, *T. brasiliensis* has shown preference for sorghum than for pigeonpea, which is similar to the known preference of the locally occurring *T. chilonis*. The statistical treatment of the data has shown that this preference, (shown by increased % parasitised eggs) for sorghum is significant in both the choice and no choice situation. This implies that the parasitism by *T. brasiliensis* is likely to be greater on sorghum, whether the pigeonpea is available for comparison or not. However, the extent of differences in the parasitism levels on the two hosts, was also significant when compared between the two tests—choice and no choice. This suggests that the extent of expression of preference for sorghum is substantially influenced by the presence or absence of the less preferred host, pigeonpea in the test environment. By these statistical procedures, we could effectively confirm the basic preference of the parasitoid, *T. brasiliensis* for sorghum (or non-preference for pigeonpea) by assessment of the response in choice and no choice situations.

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APPENDIX I

The m_o l. e of β, σ^2 and ρ (under no restriction) are given by the equations :

$$\partial L_1 / \partial \beta = 2\,(X'V^{-1}Y - X'V^{-1}X\beta) \,/\, 2\sigma^2 = 0 \quad (1)$$

$$\partial L_1 / \partial \sigma^2 = 2n / (\sigma^2) + (Y - X\beta)'\, V^{-1}\, (Y - X\beta) / (2\sigma^4) = 0 \quad (2)$$

$$\partial L_1 / \partial \rho = 0 \quad (3)$$

where $\partial L_1/\partial\beta = (\partial L_1/\partial\zeta_p\ '\ \partial L_1/\partial\zeta_s\ ,\ \partial L_1/\partial\delta)'$ is a column vector of partial derivatives of L_1 with respect to the elements ζ_p , τ_s , δ of the vector β. The definition and some results on the vector differentiation of scalar functions can be seen in Rao (1973) pages 71-72.

Further, we note the following :

$$Y'V^{-1}X\beta = n\,[\ \{(1-\rho^2)\,Y_1 + Y_3 - \rho Y_4\}\ \tau_p + \{1-\rho^2)\,Y_2 - \rho Y_3 + Y_4)\ \tau_s +$$

$$(1+\rho)\,(Y_3 - Y_4)\,\delta]\ /\ (1-\rho^2)$$

$$\beta' X V^{-1} X\beta = n\ [\,(2-\rho^2)\ \tau_p^2 + \tau_s^2) + 2\,(1+\rho)\,\delta^2 - 2\rho\tau_p\ \tau_s + 2\ (1+\rho)\,\delta\,(\tau_p - \tau_s)\,]$$

$$/\ (1-\rho^2)$$

Equation (3) can be written as

$$\partial L_1 / \partial\rho = n\rho^2 / (1-\rho^2) - (\partial\,(w(\rho)\,/\,(1-\rho^2))\,/\,\partial\rho\,)\,/\,(2\sigma^2)$$

$$= \rho\,/\,(1-\rho^2) - \{(1-\rho^2)\,\partial w(\rho)\,/\,\partial\rho + 2\rho\, w(\rho)\,\}\ /\ (2\sigma^2\,(1-\rho^2)^2$$

$$= 0$$

where the function :

$$w(\rho) = SS_{33} + SS_{44} - 2\rho SS_{34} - 2n\,(\,(Y_3 - \rho Y_4\,)\,\tau_p + (Y_4 - \rho Y)\,\tau_s$$

$$+ (1+\rho)\,(Y_3 - Y_4\,)\,\delta) +$$

$$+n\,\{\,(2-\rho^2)\,(\tau_p^2 + \tau_s^2) + 2\,(1+\rho)\,\delta^2 - 2\rho\tau_s\ \tau_p + 2\ (1+\rho)\delta(\tau_p - \tau_s\,)\,\} \qquad (4)$$

and its derivative,

$$\partial w(\rho)/\partial\rho \cong -2SS_{34} + 2n[Y_4\ \tau_p + 2Y_3\ \tau_s - \delta\,(Y_3 - Y_4\,) - p(\tau_p^2 + \tau_s^2)$$

$$+\delta^2 - \tau_p\ \tau_s + \delta\ (\tau_p - \tau_s\,)\ \}\,]$$

APPENDIX II

For the sake of completeness, the Newton-Raphson method is described to solve the equation f(x) = 0.

Let f' (x) = d f (x) / dx, be the first derivative of f(x).

Furthermore, suppose (guess) initial value for x is x_0

Then compute the next value for x, given by $x_1 = x^0 - f(x_0) / f'(x_0)$.

Treat x_1 as initial value and compute $x_2 = x_1 - f(x_1) / f'(x_1)$ and iterate the process till the differences between two consecutive values x_n and x_{n+1} are as close as desired.

In order to solve equations (4.3)—(4.5), guess

a value of ρ, say, ρ_0 (step 1). Compute $\widehat{\beta}_m$ and $\widehat{\sigma}_m^2$ with $\rho = \rho_0$. (step 2). Solve for $g(\rho) = 0$ using Newton-Rasphon method (or otherwise) say, $\rho_m = \rho_1$. Go to step 1 and step 2 with $\rho = \rho_1$ and repeat this process of iteration until a desired convergence is achieved.

APPENDIX III

The asymptotic variance—covariance matrix V_a of $\widehat{\rho}_m$, $\widehat{\sigma}_m$, $\widehat{\beta}_m$ is the inverse of the Fisher's information matrix given by

$$M = -\begin{pmatrix} E(\partial^2 L_1/\partial\rho^2) & E(\partial^2 L_1) / \partial\rho\partial\sigma^2) & E(\partial^2 L_1/\partial\rho\partial\beta) \\ & E(\partial^2 L_1 / \partial(\sigma^2)^2) & E(\partial^2 L_1/\partial\sigma^2\partial\beta) \\ \text{Symm.} & & E(\partial^2 L_1/\partial\beta\partial\beta) \end{pmatrix} \quad (5)$$

where $\partial^2 L_1/\partial\rho\partial\beta = (\partial^2 L_1 / \partial\rho\partial\tau_p, \partial^2 L_1/\partial\rho\partial^2{}_s, \partial^2 L_1/\partial\rho\partial\delta)$

$$\partial^2 L_1 / \partial\beta\partial\beta = \begin{pmatrix} \partial^2 L_1 / \partial^2\tau_p & \partial^2 L_1 / \partial\tau_p\,\partial\tau_s & \partial^2 L_1/\partial\tau_p\,\partial\delta \\ & \partial^2 L_1/\partial\tau_s^2 & \partial^2 L_1/\partial\tau_s\,\partial\delta \\ \text{Symm.} & & \partial^2 L_1/\partial\delta^2 \end{pmatrix}$$

Let us evaluate the various expectations in expression (5) in the following.

$$\partial^2 L_1/\partial\rho^2 = \partial(\partial L_1/\partial\rho) / \partial\rho$$

$$= \partial \{ n\rho / (1-\rho^2) - \partial w(\rho) / \partial\rho/(2\sigma^2(1-\rho^2)) - \rho w(\rho) / (\sigma^2(1-\rho^2)^2 \} / \partial\rho$$

$$= (1+\rho^2) n / (1-\rho^2)^2 - \partial^2 w(\rho)/\partial\rho^2/(2\sigma^2(1-\rho^2)) - 2\rho\partial w(\rho) / \partial\rho/\sigma^2(1-\rho^2)^2$$
$$-(1+3\rho^2) w(\rho) / \sigma^2 (1-\rho^2)^3$$

where $\partial w(\rho) / \partial \rho$ is as in (4) and (6)

$$\partial^2 w(\rho) / \partial \rho^2 = 2n(\tau_p^2 + \tau_s^2)$$

$$\partial^2 L_1/\partial \rho \partial \sigma^2 = \{ (1-\rho^2) \partial w(\rho) / \partial \rho + 2\rho w(\rho) \} / (2 \sigma^4 (1-\rho^2)^2 \tag{7}$$

$$\partial^2 L_1/\partial \rho \partial \beta = (\partial (X V^{-1} Y X'V^{-1} X\beta) / \partial \rho) / \sigma^2 \tag{8}$$

$$\partial^2 L_1/\partial (\sigma^2)^2 = 2n/\sigma^4 - (Y - X\beta)' V^{-1}(Y - X\beta) / \sigma^6 \tag{9}$$

$$\partial^2 L_1/\partial \sigma^2 \partial \beta = -(X'V^{-1} Y - X'V^{-1} X\beta) / \sigma^6 \tag{10}$$

$$\partial^2 L_1/\partial \beta \partial \beta = -(X V^{-1} X) / \sigma^2 \tag{11}$$

The expectation of the expressions in (6) to (11) are simplified as in the following.

We find after some simplification

$$E(w(\rho)) = -(1-\rho^2)n(2\sigma^2 + \tau_p^2 + \tau_s^2)$$

$$E(\partial w(\rho)) = -2\rho n(\sigma^2 + \tau_p^2 + \tau_s^2) \tag{12}$$

$$E(\partial^2 w(\rho) / \partial \rho^2) = -2n(\tau_p^2 + \tau_s^2)$$

Equation (6) using (12) yields the expression for $E(\partial^2 L_1 / \partial \rho^2)$

$$E(\partial^2 L_1 / \partial \rho^2) = -(2 + \rho^2)n / (1-\rho^2)^2$$

$$E(\partial^2 L_1 / \partial \rho \partial \sigma^2) = \rho n / \sigma^2 (1-\rho^2)$$

$$E(\partial^2 L_1 / \partial \rho \partial \beta) = \partial / \partial \rho \; E(X V^{-1} Y - X Y^{-1} X\beta) / \sigma^2$$

$$= 0$$

(By taking expectation under differential sign and using $E(Y) = X\beta$.)

$$E(\partial^2 L_1 / \partial \sigma^2 \beta) = 0$$

$$E(\partial^2 L_1 / \partial \beta \partial \beta) = -(X'V^{-1}X) \sigma^2$$

In order to evaluate $E(\partial^2 L_1 / \partial(\sigma^2)^2)$ we consider the following lemma, in John (1971), page 29. Lemma : If x is a random vector with $E(\mathbf{x}) = 0$ $D(\mathbf{x}) = \Sigma$ then the expected value of the quadratic form $\mathbf{x}' A \mathbf{x}$ is given by

$$E(x' Ax) = \text{trace}\,(A\Sigma)$$

Thus by putting $\mathbf{x} = Y - X\beta$, $\Sigma = \sigma^2 V$ we obtain

$$E(\partial^2 L_1 / \partial(\sigma^2)^2) \quad 2 / \sigma^4$$

The information matrix M for $(\rho, \sigma^2, \tau_p, \tau_s, \delta)$ is

$$M = -\begin{vmatrix} (2+\rho^2)n/(1-\rho^2)^2 & -\rho n/\sigma^2(1-\rho^2) & 0 \\ & 2n/\sigma^4 & 0' \\ \text{Symm.} & & (1/\sigma^2) X V^{-1} X \end{vmatrix}$$

$\mathbf{O}'$ is row vector of order three with zeroes.